

Name: _____

Section: _____

Last 4 digits of student ID #: _____

This exam has ten multiple choice questions (five points each) and five free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

1. You must give your *final answers* in the *multiple choice answer box* on the front page of your exam.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box*.

On the free response problems:

1. Clearly indicate your answer and the reasoning used to arrive at that answer (*unsupported answers may not receive credit*),
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Multiple Choice Answers

| Question | | | | | |
|----------|---|---|---|---|---|
| 1 | A | B | C | D | E |
| 2 | A | B | C | D | E |
| 3 | A | B | C | D | E |
| 4 | A | B | C | D | E |
| 5 | A | B | C | D | E |
| 6 | A | B | C | D | E |
| 7 | A | B | C | D | E |
| 8 | A | B | C | D | E |
| 9 | A | B | C | D | E |
| 10 | A | B | C | D | E |

Exam Scores

| Question | Score | Total |
|----------|-------|-------|
| MC | | 50 |
| 11 | | 10 |
| 12 | | 10 |
| 13 | | 10 |
| 14 | | 10 |
| 15 | | 10 |
| Total | | 100 |

Record the correct answer to the following problems on the front page of this exam.

1. Which of the following is an equation of the tangent line to the curve given by

$$x^2 + \sin y = xy^2 + 1$$

at the point $(1, 0)$?

- A. $y = -2x + 2.$
- B. $y = -x + 1.$
- C. $y = 0.$
- D. $y = -2x - 2.$
- E. $y = 2x - 2.$

2. Let $a > 0$ be a constant. Consider the function

$$f(x) = \ln(x^2 + 2ax + a^2 + 1)$$

on the interval $(-\infty, \infty)$. Find the maximal open interval on which f is decreasing.

- A. $(0, \infty).$
- B. $(-\infty, -a).$
- C. $(-a, \infty).$
- D. $(a, \infty).$
- E. The function is nowhere decreasing.

Record the correct answer to the following problems on the front page of this exam.

3. Let

$$f(x) = x \tan x + e^{2 \sin x}.$$

Find $f'(0)$.

- A. e
- B. -1
- C. $\pi/2$
- D. 2
- E. -2

4. Evaluate the following limit

$$\lim_{x \rightarrow 0} \frac{\ln(1+x) - x}{x^2}.$$

- A. $-\frac{1}{2}$
- B. $\frac{1}{2}$
- C. 0
- D. $\ln 2$
- E. The limit does not exist.

Record the correct answer to the following problems on the front page of this exam.

5. Find the linearization of $f(x) = \frac{x}{1+x^2}$ at $x = 2$.

A. $L(x) = \frac{2}{5}$

B. $L(x) = -\frac{3}{25}x + \frac{16}{25}$

C. $L(x) = \frac{3}{25}x - \frac{1}{25}$

D. $L(x) = x + 2$

E. $L(x) = \frac{2}{5}x$

6. At a given moment, a plane passes directly above a radar station at an altitude of 6 *km*. The plane's speed is 480 *km/h*. How fast is the distance between the plane and the station changing one minute later?

A. 384 *km/h*

B. $240\sqrt{2}$ *km/h*

C. $240\sqrt{3}$ *km/h*

D. $480\sqrt{2}$ *km/h*.

E. $480\sqrt{3}$ *km/h*

Record the correct answer to the following problems on the front page of this exam.

7. Which of the following statements is NOT true?

- A. If $f(x)$ is continuous on $[a, b]$, then f attains an absolute minimum on the interval.
- B. If $f'(x) = g'(x)$ for all $x \in (a, b)$, then $f(x) = g(x) + C$ for some constant C .
- C. If $\int_0^1 f(x) dx \geq 0$, then $f(x) \geq 0$ for all $x \in [0, 1]$.
- D. If f is continuous on $[0, 1]$ and $f(0) \neq f(1)$, then there exists $c \in (0, 1)$ such that

$$f(c) = \frac{1}{2}(f(0) + f(1)).$$

- E. If f is differentiable at a , then $\lim_{x \rightarrow a} f(x) = f(a)$.

8. Suppose that $f'(x) = \frac{1}{x^2+4}$ for all $x > 0$ and $f(2) = 0$. Find $f(2\sqrt{3})$.

- A. $\frac{\pi}{2}$
- B. $\frac{\pi}{3}$
- C. $\frac{\pi}{6}$
- D. $\frac{\pi}{24}$
- E. 1

Record the correct answer to the following problems on the front page of this exam.

9. A particle is traveling along a straight line with a velocity of

$$v(t) = t^2 - 7t + 6 \text{ meters/minute.}$$

What is the particle's total distance traveled during the time interval $[0, 4]$?

- A. $\frac{49}{3}$ meters
- B. $\frac{91}{6}$ meters
- C. 15 meters
- D. 16 meters
- E. $\frac{32}{3}$ meters

10. Determine the integral

$$\int \frac{x^2\sqrt{x} + 2}{x} dx.$$

- A. $\frac{2}{7}x^3\sqrt{x} + 2x + C.$
- B. $\frac{2}{5}x^2\sqrt{x} + 2\ln x + C.$
- C. $\frac{2}{7}x^3 + 2x - \ln x + C.$
- D. $2x^2\sqrt{x} + \ln x + C.$
- E. $3x^2\sqrt{x} - 2x + C.$

Free Response Questions: Show your work!

11. A rectangular box of height h with square base of length b has volume $V = 4$ cubic meters. Two of the side faces are made of material costing \$40 per square meter. The remaining sides, and the top and bottom, cost \$20 per square meter. Which values of b and h minimize the cost of the box?

Free Response Questions: Show your work!

12. (a) How long will it take for \$4,000 to double in value if it is deposited in an account earning 7% interest, compounded continuously? Recall that the value of the account after t years is $P(t) = P_0e^{rt}$.

- (b) After winning \$5 million in the state lottery, Jessica learns that she will receive five yearly payments of \$1 million beginning immediately. What is the PV of Jessica's prize, if $r = 6\%$? Recall that the present value (PV) of P dollars received at time t is Pe^{-rt} .

Free Response Questions: Show your work!

13. Evaluate the following integrals.

(a)

$$\int_0^{\frac{1}{2}} (12y^2 + 6y) dy$$

(b)

$$\int_{-1}^2 \sqrt{5x + 6} dx$$

Free Response Questions: Show your work!

14. (a) Suppose that f is continuous on the interval $[0, 2]$ and

$$\int_0^x f(t) dt = x^2 + x \quad \text{for } 0 \leq x \leq 2.$$

Find $f(1)$.

- (b) Find the derivative of the function g defined by

$$g(x) = \int_2^{\sqrt{x}} \frac{\cos(t^2)}{1+t} dt.$$

Free Response Questions: Show your work!

15. Compute the area of the region enclosed by the graphs of the two functions

$$f(x) = 20 + x - x^2 \quad \text{and} \quad g(x) = x^2 - 5x.$$

Present also a sketch of the two graphs.