MA 113 Calculus I — Spring 2014
Exam 4 — Wednesday, 7 May 2014

Name: ______________________

Section: ______________________

Last 4 digits of student ID #: ________

This exam has ten multiple choice questions (five points each) and five free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

- Select your answer by placing an X in the appropriate square of the multiple choice answer box on the front page of the exam.
- Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box.*

On the free response problems:

- Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
- Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E

[A,B,E,B,E, C,D,C,A,B]

Exam Scores

Question	Score	Total
MC		50
11		10
12		10
13		10
14		10
15		10
Total		100

Record the correct answer to the following problems on the front page of this exam.

1. Find the linearization of $f(x) = e^{x^2-2x}$ at $x = 2$.

   (A) $L(x) = 2x - 3$

   (B) $L(x) = 2x - 5$

   (C) $L(x) = 2x - 1$

   (D) $L(x) = -2x + 5$

   (E) $L(x) = -2x + 3$

2. A particle moves with constant acceleration of 5 meters/second^2. If the velocity at time $t = 4$ seconds is 3 meters/second, find the velocity at time $t = 2$ seconds.

   (A) -10 meters/second

   (B) -7 meters/second

   (C) -2 meters/second

   (D) 10 meters/second

   (E) 13 meters/second

Record the correct answer to the following problems on the front page of this exam.

3. Find A so that the function $f(x) = \begin{cases} e^x, & x > 1 \\ Ax, & x \leq 1 \end{cases}$ is continuous for all x.

   (A) $A = -1$
   (B) $A = 0$
   (C) $A = 1/e$
   (D) $A = 1$
   (E) $A = e$

4. If $f(x) = 3x + 1$ and g is the inverse function, find the derivative $g'(x)$.

   (A) $g'(x) = 1/(3x + 1)$
   (B) $g'(x) = 1/3$
   (C) $g'(x) = -1/3$
   (D) $g'(x) = 3$
   (E) $g'(x) = -3$

Record the correct answer to the following problems on the front page of this exam.

5. Suppose $f(1) = 2$ and $f'(1) = 3$. Set $g(x) = f(x^2)$ and find $g'(1)$.

   (A) 2
   (B) 3
   (C) 4
   (D) 5
   (E) 6

6. Suppose $f(x) = \dfrac{x^2 + Ax + 1}{\sin(x-1)}$. There is exactly one value of A for which we can apply L'Hopital's rule to compute the limit $L = \lim_{x\to 1} f(x)$. Find this value of A and compute the limit.

   (A) $L = -2$
   (B) $L = -1$
   (C) $L = 0$
   (D) $L = 1$
   (E) $L = 2$

Record the correct answer to the following problems on the front page of this exam.

7. Find the value of c for which the function $f(x) = x^3 + 3x^2 + cx + 2014$ has a critical point, but does not have a local maximum or minimum.

   (A) $c = 0$

   (B) $c = 1$

   (C) $c = 2$

   (D) $c = 3$

   (E) $c = 4$

8. Find the absolute minimum value of the function $f(x) = \sqrt{|x|}$ on the interval $[-1, 2]$.

   (A) $-\sqrt{2}$

   (B) -1

   (C) 0

   (D) 1

   (E) $\sqrt{2}$

Record the correct answer to the following problems on the front page of this exam.

9. Consider the curve defined by the equation $x^2 + 2y^2 = 9$. Find dy/dx at the point where $x = 1$ and $y > 0$.

   (A) $-1/4$

   (B) $-1/2$

   (C) 0

   (D) $1/4$

   (E) $1/2$

10. The figure below gives the graphs of three functions, g, h, and k. These are the graph of a function f and its first two derivatives, f' and f''. Determine which is f, f', and f''.

    (A) $f = g,\ f' = h,\ f'' = k$

    (B) $f = g,\ f' = k,\ f'' = h$

    (C) $f = k,\ f' = g,\ f'' = h$

    (D) $f = k,\ f' = h,\ f'' = g$

    (E) $f = h,\ f' = k,\ f'' = g$

Free Response Questions: Show your work!

11. We construct a rectangular pen. One pair of parallel sides are made of wooden fence that costs \$10 per meter and the other pair of opposite sides are built of wire fence that costs \$5 per meter. If the area of the pen is to be 200 square meters, give the cost of the pen with least cost. Use calclulus to explain why you know you have found the least expensive pen.

Let x, y length and width of pen in meters. $Cost = 2x(10) + 2y(5) = 20x + 10y$	2 points cost function
$Area = xy = 200$ meters^2, $y = 200/x$	1 point constraint equation
$C(x) = 20x + \frac{2000}{x}$	1 point substitution
$x > 0$	1 point domain
$C'(x) = 20 - \frac{2000}{x^2} = 0.$ $x^2 = \frac{2000}{20} = 1000, \quad x = \pm 10.$ The only critical point in domain is $x = 10$	2 points find critical points
$C''(x) = \frac{4000}{x^3} > 0$ whenever $x > 0$, then $C(x)$ is concave up for $x > 0$ and $C(10)$ is a minimum.	2 points 2nd derivative test
Minimum cost is $C(10) = 400$ dollars.	1 point final answer

Free Response Questions: Show your work!

12. Evaluate the following integrals. Show your work.

(a) $\displaystyle\int_1^4 \frac{1}{(1+x)^2}\,dx$

(b) $\displaystyle\int_1^2 x\cos(\frac{\pi}{2}x^2)\,dx$

(c) $\displaystyle\int x\sqrt{x+3}\,dx$

(a) $u = 1+x, \qquad du = dx$ — 3 points

$$\int_1^4 \frac{1}{(1+x)^2}\,dx = \int_2^5 \frac{du}{u^2} = -\frac{1}{u}\Big|_2^5 = \frac{-1}{5}+\frac{1}{2} = \frac{3}{10}$$

(b) $u = \frac{\pi}{2}x^2, \qquad du = \pi x\,dx$ — 3 points

$$\int_1^2 x\cos\left(\frac{\pi}{2}x^2\right)dx = \frac{1}{\pi}\int_{\pi/2}^{2\pi}\cos u\,du = \frac{1}{\pi}\sin u\Big|_{\pi/2}^{2\pi} = \frac{1}{\pi}\left(\sin(2\pi)-\sin(\frac{\pi}{2})\right) = \frac{-1}{\pi}$$

(c) $u = x+3, \qquad du = dx$ — 4 points

$$\int x\sqrt{x+3}\,dx = \int (u-3)u^{1/2}\,du = \int u^{3/2}-3u^{1/2}\,du = \frac{2}{5}u^{5/2}-2u^{3/2}+C = \frac{2}{5}(x+3)^{5/2}-2(x+3)^{3/2}+C$$

13. A population P is growing at a rate of 2% per year.

    (a) If $P(t)$ is the population at time t years, express P' in terms of P.

    (b) Suppose that the population at time $t = 1$ years is 100. Find the population $P(t)$ for all times t.

    (c) Find the population at time $t = 10$ years. Round your answer to the nearest whole number.

(a) $$P' = 0.02P$$	4 points
(b) $$P(t) = 100e^{0.02(t-1)}$$ or $$P(t) = \frac{100}{e^{0.02}}e^{0.02t}$$	4 points
(c) $$\begin{aligned} P(10) &= 100e^{(0.02)9} \\ &= 119.72 \\ &\approx 120. \end{aligned}$$	2 points

Free Response Questions: Show your work!

14. (a) Sketch the graphs of $f(x) = x^2 - 2x - 3$ and $g(x) = x + 1$ on the same axes and label the graphs.

    (b) Express the area of the region enclosed by the graphs as an integral.

    (c) Find the area of the region enclosed by these graphs.

(a)	2 points
(b) $$\int_{-1}^{4} g(x) - f(x)\,dx = \int_{-1}^{4} (x+1) - (x^2 - 2x - 3)\,dx$$	5 points total: - 2 points correct limits of integration - 3 points correct integrand
(c) $$\int_{-1}^{4} -x^2 + 3x + 4\,dx = \frac{-1}{3}x^3 + \frac{3}{2}x^2 + 4x\Big|_{-1}^{4} = \frac{125}{6} \quad \text{or} \quad 20.8\bar{3}.$$	3 points

Free Response Questions: Show your work!

15. Let $F(x) = \int_0^x e^{-t^2}\, dt$.

    (a) State part 2 of the Fundamental Theorem of Calculus.

    (b) Use part 2 of the Fundamental Theorem of Calculus to determine the derivative of F.

    (c) Find the largest open interval on which F is concave down.

(a) Let f be continuous on an open interval I and $a \in I$. Then $\int_a^x f(t)\, dt$ is differentiable, and $$\frac{d}{dx}\int_a^x f(t)\, dt = f(x).$$	4 points
(b) $$F'(x) = e^{-x^2}$$	2 points
(c) $$F''(x) = -2xe^{-x^2}$$ Then $F''(x) < 0$ when $x > 0$. Interval is $(0, \infty)$.	4 points total: - 2 points calculate $F''(x)$ - 2 points correct interval