MA 113 Calculus I Exam 4

Spring 2015 Tuesday, 5 May 2015

Name: _____

Section: _

Last 4 digits of student ID #: ____

This exam has ten multiple choice questions (five points each) and five free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

- 1. You must give your *final answers* in the *multiple choice answer box* on the front page of your exam.
- 2. Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box*.

On the free response problems:

- 1. Clearly indicate your answer and the reasoning used to arrive at that answer (unsupported answers may not receive credit),
- 2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Multiple Choice Answers

Question					
1	A	В	С	D	Е
2	А	В	С	D	Е
3	A	В	С	D	Е
4	A	В	С	D	Е
5	A	В	С	D	Е
6	A	В	С	D	Е
7	A	В	С	D	Е
8	A	В	С	D	Е
9	A	В	С	D	Е
10	А	В	С	D	Е

Exam Scores

Question	Score	Total
MC		50
11		10
12		10
13		10
14		10
15		10
Total		100

Record the correct answer to the following problems on the front page of this exam.

- 1. Find the equation of the tangent line to the graph of $f(x) = e^{2x}$ at the point x = 3.
 - (A) $y = 2e^6x e^6$
 - (B) $y = e^6 x 2e^6$
 - (C) $y = 2e^6x + e^6$
 - (D) $y = e^6 x + 2e^6$
 - (E) $y = 2e^6x 5e^6$

2. Find
$$f'(3)$$
 if $f(x) = \frac{2x}{\sqrt{1+x}}$.
(A) $\frac{3}{8}$
(B) $\frac{4}{8}$
(C) $\frac{5}{8}$
(D) $\frac{6}{8}$
(E) $\frac{7}{8}$

- 3. Suppose $f(x) = \sin(x)[g(x)]^3$, $g(\frac{\pi}{4}) = 2$, and $g'(\frac{\pi}{4}) = 3$. Find $f'(\frac{\pi}{4})$.
 - (A) $20\sqrt{2}$
 - (B) $22\sqrt{2}$
 - (C) $24\sqrt{2}$
 - (D) $26\sqrt{2}$
 - (E) $28\sqrt{2}$

Record the correct answer to the following problems on the front page of this exam.

- 4. Find $\lim_{x \to 0} \frac{\sin(3x)}{e^{2x} 1}$. (A) 0 (B) 3 (C) $\frac{2}{3}$ (D) $\frac{3}{2}$

 - (E) The limit does not exist.

5. Suppose that a function f is defined by

$$f(x) = \begin{cases} 5\sqrt{x}, & 0 < x < 4\\ cx - 2, & x = 4\\ x^2 - 6, & x > 4 \end{cases}$$

For what choice of c is f continuous at x = 4?

- (A) 1
- (B) 2
- (C) 3
- (D) 4
- (E) None of the above

Record the correct answer to the following problems on the front page of this exam.

6. Evaluate
$$\int_{-2}^{1} (-3+2|x|) dx$$
.
(A) -4
(B) -2
(C) 0
(D) 2

(E) 4

7. Suppose
$$G(x) = \int_{1}^{x^{3}} \ln(t) dt$$
. Find $G'(2)$.
(A) $27 \ln(2)$

- (B) $36\ln(2)$
- (C) $48\ln(2)$
- (D) $56\ln(2)$
- (E) $82\ln(2)$

Record the correct answer to the following problems on the front page of this exam.

- 8. The number of bacteria on a surface is increasing exponentially and at time t = 0 is increasing at the rate of .92 per minute. Given that P(0) = 1, how many bacteria are there 10 minutes later and at what rate is is the number of bacteria increasing?
 - (A) P(10) = 9897.1, P'(10) = 9105.4 per minute
 - (B) P(10) = 9105.4, P'(10) = 9897.1 per minute
 - (C) P(10) = 10758.7, P'(10) = 9897.1 per minute
 - (D) P(10) = 9105.4, P'(10) = 8376.93 per minute
 - (E) P(10) = 8376.9, P'(10) = 9105.4 per minute
- 9. Find $\int \frac{x+3}{\sqrt{(x+4)^3}} dx$. (Suggestion: Use a substitution with a good choice for *u*.) (A) $\frac{2}{3}\sqrt{x^3} + 2x + C$ (B) $2\sqrt{x+4} + \frac{2}{\sqrt{x+4}} + C$ (C) $2\sqrt{x+4} - \frac{2}{\sqrt{x+4}} + C$ (D) $\frac{1}{2}(x+3)^2 + \frac{1}{2}\frac{1}{\sqrt{x+4}} + C$ (E) $2\sqrt{x+3} + \frac{2}{\sqrt{x+3}} + C$
- 10. The volume of a sphere is changing at the rate of 24 cubic centimeters per second. Find the rate in centimeters per second at which the radius of the sphere is changing when r = 2. (The volume V of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.)
 - (A) $\frac{3}{2\pi}$ (B) $\frac{2}{\pi}$ (C) $\frac{5}{2\pi}$ (D) $\frac{3}{\pi}$ (E) $\frac{7}{2\pi}$

- 11. Consider the functions $f(x) = 27 x^2$ and $g(x) = 2x^2$.
 - (a) Sketch and label the graphs of f and g.

(b) Find the intersection points of f and g.

(c) Find the area of the region bounded by f(x) and g(x) between the vertical lines x = 0 and x = 4.

- 12. A rectangle lies in the first quadrant with one side on the positive x-axis, one side on the positive y-axis, one corner at the origin and its opposite corner lies on the curve given by $y = 1 + \frac{4}{x^2}$.
 - (a) Sketch the curve $y = 1 + \frac{4}{x^2}$ and this rectangle.

(b) Express the perimeter of the rectangle as a function of a single variable.

(c) Find the dimensions of the rectangle that has the smallest perimeter. Justify your answer.

13. Evaluate

(a)
$$\int_0^2 \frac{3x}{\sqrt{1+x^2}} \, dx$$

(b)
$$\int \left(x^2 e^{x^3} + \frac{2x^3 + 5x}{x^4 + 5x^2}\right) dx$$

- 14. Assume that the **derivative** of a function f(x) satisfies $f'(x) = x^3 6x^2 + 9x$.
 - (a) Find the intervals over which f is increasing, the intervals where f is decreasing, and find all the local minima and maxima of f.

(b) Find the intervals over which f is concave down, the intervals over which f is concave up, and find all points of inflection of f.

- 15. The velocity of an object is v(t) = 2t 6 during the time interval [2, 4].
 - (a) What is the antiderivative of v(t)?

(b) What is the displacement of the object during the given time interval?

(c) What is total distance traveled by the object during the given time interval?