Solutions

MA 113 Calculus I — Spring 2017

Exam Final — Monday, May 1, 2017

Name: ____________________

Section: ____________________

Last 4 digits of student ID #: ________

This exam has five true/false questions (two points each), ten multiple choice questions (five points each) and four free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the true/false and multiple choice problems:

1. You must give your *final answers* in the *front page answer box* on the front page of your exam.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the *front page answer box.*

On the free response problems:

1. Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

True/False		
1	T	F
2	T	F
3	T	F
4	T	F
5	T	F

Multiple Choice					
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E

A and B are the same...

Overall Exam Scores

Question	Score	Total
TF		10
MC		50
16		10
17		10
18		10
19		10
Total		100

Free Response Questions: Show your work!

16. Find the value of the following integrals.

(a) $\int_1^3 \frac{7}{(1+3x)^4}\, dx = (*)$ Set $u=1+3x$, so $du=3dx$.

Then $(*) = \frac{1}{3}\int_1^3 \frac{7}{(1+3x)^4} \cdot 3dx = \frac{1}{3}\int_4^{10} \frac{7}{u^4}\, du = \frac{7}{3}\int_4^{10} u^{-4} du$

$= \frac{7}{3}\left[\frac{u^{-3}}{-3}\right]_4^{10} = \frac{7}{3}\left[\frac{10^{-3}}{-3}\right] - \frac{7}{3}\left[\frac{4^{-3}}{-3}\right].$

(b) $\int 3\,x^5\sqrt{1+x^3}\, dx = (*)$ Set $u = 1+x^3$, so $du = 3x^2 dx$
and $x^3 = u-1$.

Then
$(*) = \int x^3 \sqrt{1+x^3}\, 3x^2 dx = \int (u-1)\sqrt{u}\, du = \int u^{3/2} - u^{1/2}\, du$

$= \frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2} + C = \frac{(1+x^3)^{5/2}}{5/2} - \frac{(1+x^3)^{3/2}}{3/2} + C.$

Free Response Questions: Show your work!

17. Suppose the velocity of a particle traveling along the x-axis is given by $v(t) = t^2 - 5t + 4$ m/sec at time t seconds. The particle is initially located 5 meters left of the origin.

(a) After 5 seconds, how far is the particle from the origin?

$s(0)=5$ where $s(t)=$ position.

So, $s(5) = 5 + \int_0^5 v(t)\,dt = 5 + \int_0^5 t^2 - 5t + 4\,dt$

$= 5 + \left[\frac{t^3}{3} - \frac{5}{2}t^2 + 4t\right]_0^5$

$= 5 + \left(\frac{5^3}{3} - \frac{5}{2}\cdot 5^2 + 4\cdot 5\right) - (0 - 0 + 0)$

$= 5 + \frac{125}{3} - \frac{125}{2} + 20$ meters.

(b) What is the total distance traveled by the particle between $t = 3$ and $t = 5$ seconds?

$v(t) = t^2 - 5t + 4 = (t-4)(t-1)$, so $v(4) = 0$.

So, $\int_3^5 |v(t)|\,dt = \int_3^4 -v(t)\,dt + \int_4^5 v(t)\,dt$

$= \left[-\frac{t^3}{3} + \frac{5}{2}t^2 - 4t\right]_3^4 + \left[\frac{t^3}{3} - \frac{5}{2}t^2 + 4t\right]_4^5$

$= \left(-\frac{4^3}{3} + \frac{5}{2}\cdot 4^2 - 4\cdot 4\right) - \left(-\frac{3^3}{3} + \frac{5}{2}3^2 - 4\cdot 3\right)$

$+ \left(\frac{5^3}{3} - \frac{5}{2}\cdot 5^2 + 4\cdot 5\right) - \left(\frac{4^3}{3} - \frac{5}{2}4^2 + 4\cdot 4\right)$ meters.

Free Response Questions: Show your work!

18. Consider the curve given by the equation $x^2y + xy^2 = x^3 + y^3$.

(a) Express dy/dx as a function of x and y.

Use implicit diff^n.

$$2xy + x^2\frac{dy}{dx} + y^2 + 2xy\frac{dy}{dx} = 3x^2 + 3y^2\frac{dy}{dx}.$$

$$\Rightarrow \quad \frac{dy}{dx}\left(x^2 + 2xy - 3y^2\right) = 3x^2 - 2xy - y^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{3x^2 - 2xy - y^2}{x^2 + 2xy - 3y^2}$$

(b) There is one point on this curve of the form $(-1, b)$ where $b > 0$. Find this point and show that it is on the curve.

Plug in $x=-1$ for equation: $y - y^2 = -1 + y^3$

$$\Rightarrow y^3 + y^2 - y - 1 = 0.$$

Note: $y = 1$ solves this, so $(-1,1)$ is the point,

and $(-1)^2\cdot(1) + (-1)\cdot(1)^2 = (-1)^3 + (1)^3$, so it is

on the curve.

(c) Find the equation of the tangent line to the curve at the point you found in the previous part. You do not need to simplify your answer.

$$y - 1 = \left(\frac{3(-1)^2 - 2(-1)(1) - 1^2}{(-1)^2 + 2(-1)(1) - 3(1)^2}\right)(x+1).$$

Free Response Questions: Show your work!

19. (a) Find the fifth-degree Taylor polynomial at $a = 0$ for $\sin(x)$. You must explain your work.

Let $f(x) = \sin x$. Then $f(0) = 0$,

$f'(0) = \cos(0) = 1$,

$f''(0) = -\sin(0) = 0$,

$f'''(0) = -\cos(0) = -1$,

$f^{(4)}(0) = \sin(0) = 0$

$f^{(5)}(0) = \cos(0) = 1$.

So, $T_5(x) = x - \dfrac{x^3}{3\cdot 2} + \dfrac{x^5}{5\cdot 4\cdot 3\cdot 2}$ using the formula for Taylor polynomials.

(b) Use the polynomial you found in part (a) to estimate the value of $\sin(1/4)$. Show your work. You do not need to simplify your answer.

$$\sin\left(\tfrac{1}{4}\right) \approx T_5\left(\tfrac{1}{4}\right) = \frac{1}{4} - \frac{\left(\frac{1}{4}\right)^3}{3\cdot 2} + \frac{\left(\frac{1}{4}\right)^5}{5\cdot 4\cdot 3\cdot 2}.$$