MA 113 Calculus I — Spring 2018

Exam 4 — Wednesday, 2 May 2018

Name: ______________________

Section: ______________________

Last 4 digits of student ID #: ________

This exam has 12 multiple choice questions (five points each) and 4 free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-buds during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

- Select your answer by placing an X in the appropriate square of the multiple choice answer box on the front page of the exam.
- Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box*.

On the free response problems:

- Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
- Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E

B, E, D, A   C, D, D, B   A, C, D, B

Exam Scores

Question	Score	Total
MC		60
13		10
14		10
15		10
16		10
Total		100

1. Find the equation of the tangent line to the graph of $f(x) = (x^2 + 9)^{3/2}$ at $x = 4$.

   (A) $y = 50x - 75$

   (B) $y = 60x - 115$

   (C) $y = 70x - 155$

   (D) $y = 80x - 195$

   (E) $y = 90x - 235$

2. Let $f(x) = \dfrac{\tan(x)}{e^{2x}}$. Find $f'(x)$. (After computing $f'(x)$, you must simplify the answer.)

   (A) $\dfrac{\tan(x) - \sec^2(x)}{e^{2x}}$

   (B) $\dfrac{2\tan(x) - \sec^2(x)}{e^{2x}}$

   (C) $\dfrac{\sec(x) - \tan(x)}{e^{2x}}$

   (D) $\dfrac{\sec^2(x) - \tan(x)}{e^{2x}}$

   (E) $\dfrac{\sec^2(x) - 2\tan(x)}{e^{2x}}$

Record the correct answer to the following problems on the front page of this exam.

3. Assume that $f''(x) = 6x - 4$, $f'(1) = 2$, and $f(2) = 10$. Find $f(x)$.

   (A) $x^3 - 2x^2 + 10$
   (B) $x^3 - 2x^2 + x + 8$
   (C) $x^3 - 2x^2 + 2x + 6$
   (D) $x^3 - 2x^2 + 3x + 4$
   (E) $x^3 - 2x^2 + 4x + 2$

4. Assume that $f'(x) = \cos(x)(x - \frac{\pi}{3})$. Over which intervals in $[0, \pi]$ is f decreasing? (Note that $f'(x)$ is given, not $f(x)$.)

   (A) $(0, \pi/3)$ and $(\pi/2, \pi)$
   (B) $(0, \pi/2)$
   (C) $(\pi/3, \pi/2)$
   (D) $(\pi/3, \pi)$
   (E) $(0, \pi)$

Record the correct answer to the following problems on the front page of this exam.

5. Suppose that $x\ln(y) + \sin(y) = 3x^2 + 4y^3$. Find $\dfrac{dy}{dx}$.

   (A) $\dfrac{dy}{dx} = \dfrac{6x - \ln(y)}{\frac{x}{y} + \cos(y) + 12y^2}$

   (B) $\dfrac{dy}{dx} = \dfrac{6x + \ln(y)}{\frac{x}{y} + \cos(y) - 12y^2}$

   (C) $\dfrac{dy}{dx} = \dfrac{6x - \ln(y)}{\frac{x}{y} + \cos(y) - 12y^2}$

   (D) $\dfrac{dy}{dx} = \dfrac{6x - \ln(y)}{\frac{x}{y} - \cos(y) - 12y^2}$

   (E) $\dfrac{dy}{dx} = \dfrac{x^3 - \ln(y)}{\frac{x}{y} + \cos(y) - y^4}$

6. If $2x + y = 9$, what is the smallest possible value of $4x^2 + 3y^2$?

   (A) 60

   (B) 60.25

   (C) 60.50

   (D) 60.75

   (E) 61

Record the correct answer to the following problems on the front page of this exam.

7. The volume of a cube with edge length x cm is given by $V(x) = x^3$ cm^3. The cube is expanding in such a way that the edge of the cube has length $x = 3e^{2t}$ at time t seconds. What is the rate of change of the volume of the cube when $t = 4$?

   (A) $162e^{12}$ cm^3/s

   (B) $324e^{12}$ cm^3/s

   (C) $162e^{24}$ cm^3/s

   (D) $324e^{24}$ cm^3/s

   (E) None of the above

8. Find the 3^{rd} order Taylor polynomial approximation of $(1+x)^{3/2}$ at $a = 0$. (Recall that the formula for the n^{th} order Taylor polynomial for $f(x)$ at $x = a$ is given by $f(a) + \sum_{k=1}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$, where $k! = (1)(2)(3)\cdots(k)$.)

   (A) $1 + \frac{3}{2}x + \frac{3}{4}x^2 - \frac{1}{16}x^3$

   (B) $1 + \frac{3}{2}x + \frac{3}{8}x^2 - \frac{1}{16}x^3$

   (C) $1 + \frac{3}{2}x + \frac{3}{8}x^2 - \frac{3}{8}x^3$

   (D) $1 + \frac{1}{2}x + \frac{3}{8}x^2 - \frac{1}{16}x^3$

   (E) $1 + \frac{3}{2}x + \frac{3}{8}x^2 + \frac{1}{16}x^3$

Record the correct answer to the following problems on the front page of this exam.

9. A particle travels in a straight line with velocity $v(t) = 3t^2 - 12t + 9$ m/s. Find the net displacement over the interval $[1, 4]$.

   (A) 0 m

   (B) 1 m

   (C) 2 m

   (D) 3 m

   (E) 4 m

10. A particle travels in a straight line with velocity $v(t) = 3t^2 - 12t + 9$ m/s. Find the total distance traveled over the interval $[1, 4]$. (Note that $3t^2 - 12t + 9 = (3t - 3)(t - 3)$.)

    (A) 6 m

    (B) 7 m

    (C) 8 m

    (D) 9 m

    (E) 10 m

Record the correct answer to the following problems on the front page of this exam.

11. Find $\displaystyle\int_{\pi/3}^{\pi} \sin(x)\, dx$.

    (A) $-1/2$

    (B) $1/2$

    (C) 1

    (D) $3/2$

    (E) $-3/2$

12. Assume that $\dfrac{dy}{dx} = 5y$. If $y(1) = 4$, then find $y(3)$.

    (A) $4e^{15}$

    (B) $4e^{10}$

    (C) $4e^{3}$

    (D) $5e^{15}$

    (E) $5e^{10}$

Free Response Questions: Show your work!

13. (a) Find the limit $\lim_{x\to 1} \frac{x^3 - 2x^2 + x}{x^3 - x^2 - x + 1}$. Justify your answer. (Students who guess the answer based on a few values of the function will not receive full credit.)

$$\lim_{x\to 1} \frac{x^3 - 2x^2 + x}{x^3 - x^2 - x + 1} = \lim_{x\to 1} \frac{3x^2 - 4x + 1}{3x^2 - 2x - 1} = \lim_{x\to 1} \frac{6x - 4}{6x - 2} = \frac{1}{2}.$$

(b) Find $\frac{d}{dx}\left(\int_a^x \sec(t)\,dt\right)$.

$$\frac{d}{dx}\left(\int_a^x \sec(t)\,dt\right) = \sec(x)$$

(c) Find $\frac{d}{dx}\left(\int_a^{x^2} \sec(t)\,dt\right)$.

$$\frac{d}{dx}\left(\int_a^{x^2} \sec(t)\,dt\right) = 2x\sec(x^2).$$

Free Response Questions: Show your work!

14. (a) Find $\int (6x^2+8)\sqrt{x^3+4x}\,dx$.

Let $u = x^3+4x$. Then $du = (3x^2+4)\,dx$. Thus $\int (6x^2+8)\sqrt{x^3+4x}\,dx = \int 2u^{1/2}\,du = \frac{4}{3}u^{3/2}+C = \frac{4}{3}(x^3+4x)^{3/2}+C$.

(b) Find $\int \left(\frac{2x^2+\frac{3}{x}}{\sqrt{x}} + e^{3x}\right) dx$.

$\int \left(\frac{2x^2+\frac{3}{x}}{\sqrt{x}} + e^{3x}\right) dx = \int \left(2x^{3/2}+3x^{-3/2}+e^{3x}\right) = \frac{4}{5}x^{5/2} - 6x^{-1/2} + \frac{1}{3}e^{3x} + C.$

Free Response Questions: Show your work!

15. (a) Find the linearization of $f(x) = \sqrt{x+7}$ at $x = 2$.

$f'(x) = \dfrac{1}{2\sqrt{x+7}}$. $L(x) = f(2) + f'(2)(x-2) = 3 + \dfrac{1}{6}(x-2)$.

(b) Use the linearization in (a) to approximate $\sqrt{8.99}$. (Show your work. No credit will be given for just using a calculator.)

$\sqrt{8.99} = f(1.99) \approx L(1.99) = 3 + \dfrac{1}{6}(-0.01) = 2.998\bar{3}$.

16. (a) Compute $\int_1^4 3xe^{x^2}\, dx$.

$$\int_1^4 3xe^{x^2}\, dx = \frac{3}{2}e^{x^2} \,|_1^4 = \frac{3}{2}(e^{16} - e).$$

(b) Compute $\int_3^5 \frac{2x+1}{x^2+x}\, dx$.

$$\int_3^5 \frac{2x+1}{x^2+x}\, dx = \ln(x^2+x) \,|_3^5 = \ln(30) - \ln(12) = \ln\left(\frac{30}{12}\right) = \ln\left(\frac{5}{2}\right).$$