MA 113 Calculus I Spring 2019
Exam ~~2~~ 4 Tuesday, ~~1 March~~ May 2019

Name: _____

Section: _____

Last 4 digits of student ID #: _____

This is a two-hour exam. This exam has 12 multiple choice questions (five points each) and 4 free response questions (ten points each). Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-buds during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

- Select your answer by placing an X in the appropriate square of the multiple choice answer box on the front page of the exam.
- Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box.*

On the free response problems:

- Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
- Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question.

Multiple Choice Answers

Question						
1	X	B	C	D	E	A
2	A	X	C	D	E	B
3	A	B	C	X	E	D
4	X	B	C	D	E	A
5	A	X	C	D	E	B
6	A	B	X	D	E	C
7	A	B	C	D	X	E
8	A	B	C	X	E	D
9	A	B	C	X	E	D
10	X	B	C	D	E	A
11	A	B	X	D	E	C
12	A	X	C	D	E	B

Exam Scores

Question	Score	Total
MC		60
13		10
14		10
15		10
16		10
Total		100

* KEY * KEY * KEY * KEY *

Free Response Questions: Show your work!

(6 pts) 15. (a) Find the linearization of $f(x) = e^{2x}$ at $a = 0$.

ypoint $f(0) = e^{2(0)} = e^0 = 1$ $(0,1)$ (+1)

Slope $f'(x) = e^{2x} \cdot 2 = 2e^{2x}$ (+1)

$f'(0) = 2e^{2(0)} = 2(1) = 2$ (+1)

$y - 1 = 2(x - 0)$ (+3) one point for plugging each number into the correct location

OR $y = 2x + 1$

(4 pts) (b) Use the linearization you found in part (a) to estimate the value of $e^{0.2}$. Show your work. No credit will be given for using only a calculator.

$e^{0.2} = e^{2(0.1)} = f(0.1)$ (+1)

So then $y = 2(0.1) + 1 = 1.2$ (+2) (+1)

* 2/4 pts for plugging in 0.2 and getting 1.4

14. (a) Find $\int \frac{1}{x\,(\ln x)^3}\,dx$.

Set $u = \ln x$, so $du = \frac{1}{x}\,dx$.

$$\int \frac{1}{(\ln x)^3}\cdot\frac{1}{x}\,dx = \int \frac{1}{u^3}\,du = \int u^{-3}\,du = \frac{u^{-2}}{-2} + C$$

$$= \frac{-1}{2\,(\ln x)^2} + C$$

(+1) for u (+1) for du (+1) antiderivative (+1) for subbing $\ln x$ back (+1)

(b) Find $\int_0^{\pi} x\cos(4x^2)\,dx$.

Set $u = 4x^2$, so $du = 8x\,dx$.

$$\frac{1}{8}\int_0^{\pi} \cos(4x^2)\cdot 8x\,dx = \frac{1}{8}\int_0^{4\pi^2} \cos(u)\,du$$

(+1) for changing bounds or for changing them back to x's.

(+1) for u (+1) for du

$$= \frac{1}{8}\Big[\sin u\Big]_0^{4\pi^2}$$

(+1) for antiderivative.

$$= \frac{1}{8}\sin(4\pi^2) \approx 0.1222925\ldots$$

(+1) for answer.

Team Readdy + Gauthier

Free Response Questions: Show your work!

13. Find the following limits. Justify your answers. (Students who guess the answer based on a few values of the function will not receive full credit.)

(a) $\lim_{x\to 0} \dfrac{x^2}{\cos(3x)-1}$ → $\dfrac{0^2}{\cos 0 - 1} = \dfrac{0}{1-1} = \dfrac{0}{0}$ ① Use L'Hôpital.

$= \lim_{x\to 0} \dfrac{2x}{-3\sin(3x)}$ ①. → $\dfrac{2\cdot(0)}{-3\sin(0)} = \dfrac{0}{0}$ ①. Use L'Hôpital again.

$= \lim_{x\to 0} \dfrac{2}{-3(3)\cos(3x)} = \dfrac{2}{-3(3)(\cos 0)} = \dfrac{2}{-3(3)(1)}$ ①.

$= -2/9$.

(b) $\lim_{x\to\infty} \dfrac{9x^8 - x^6 + x - 12}{7x^8 + x^5 - x^2 + 8}$ · $\dfrac{\frac{1}{x^8}}{\frac{1}{x^8}}$ ①

$= \lim_{x\to 0} \dfrac{9 - \frac{1}{x^2} + \frac{1}{x^7} - \frac{12}{x^8}}{7 + \frac{1}{x^3} - \frac{1}{x^6} + \frac{8}{x^8}}$ ① ① = $\dfrac{9-0+0-0}{7+0-0+0} = \dfrac{9}{7}$. ①.

or...

Use L'Hôpital. 8 times. (ouch). → $\dfrac{24192}{18816}$ $\left(\dfrac{362880}{282240}\right)$ $(= 9/7)$.

2 pts if use L'Hôp twice & then give up.

or...

logic/leading coeff rule.

KEY

(-2) if sum/diff/dist

Free Response Questions: Show your work!

16. A particle moves in a straight line so that its velocity at time t seconds is

$$v(t) = t(t-2)(t-4) = t^3 - 6t^2 + 8t.$$

meters per second.

(a) Find the displacement of the particle during the time interval $1 \le t \le 3$. Include units!

$$\text{Displacement} = \int_1^3 v(t)\,dt$$ ① integral of $v(t)$, with correct bounds

$$= \int_1^3 t^3 - 6t^2 + 8t\,dt$$

$$= \left[\frac{t^4}{4} - 2t^3 + 4t^2\right]_1^3$$ ① antiderivative

$$= \left(\frac{81}{4} - 54 + 36\right) - \left(\frac{1}{4} - 2 + 4\right)$$ ① FTC 2

$$= \frac{9}{4} - \frac{9}{4}$$

$$= \boxed{0\text{m}}$$ ① arithmetic and answer

(b) Find the total distance traveled by the particle during $1 \le t \le 3$. Include units!

$$\text{Distance} = \int_1^3 |v(t)|\,dt = \int_1^2 v(t)\,dt + \int_2^3 -v(t)\,dt$$

① integral of $|v(t)|$ (could be implicit)

② split into two integrals with correct bounds and sign change

① antiderivative and FTC 2

① arithmetic and answer

$$= \left[\frac{t^4}{4} - 2t^3 + 4t^2\right]_1^2 - \left[\frac{t^4}{4} - 2t^3 + 4t^2\right]_2^3$$

$$= \left(\left(4 - 16 + 16\right) - \left(\frac{1}{4} - 2 + 4\right)\right)$$

$$- \left(\left(\frac{81}{4} - 54 + 36\right) - \left(4 - 16 + 16\right)\right)$$

$$= \frac{7}{4} - -\frac{7}{4}$$

$$= \frac{7}{2} = \boxed{3.5\text{m}}$$

① units correct in both parts (take off if either part has missing or incorrect units)