Exam 4

Form A

Name: _____ Section and/or TA: _____

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 14 multiple choice questions and 3 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems.

Multiple Choice Questions

1	A	B	C	D	E	**8**	A	B	C	D	E
2	A	B	C	D	E	**9**	A	B	C	D	E
3	A	B	C	D	E	**10**	A	B	C	D	E
4	A	B	C	D	E	**11**	A	B	C	D	E
5	A	B	C	D	E	**12**	A	B	C	D	E
6	A	B	C	D	E	**13**	A	B	C	D	E
7	A	B	C	D	E	**14**	A	B	C	D	E

SCORE

Multiple Choice	15	16	17	Total Score
70	10	10	10	100

Trigonometric Identities

$$\sin^2(x) + \cos^2(x) = 1$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

Multiple Choice Questions

1. Find the values of A and B such that the function

$$f(x) = \begin{cases} -2x^2 + 5, & x \leq -1 \\ Ax + B, & -1 < x < 2 \\ 2x^2 - 3, & 2 \leq x \end{cases}$$

is continuous.

A. $A = \frac{2}{3}$, $B = \frac{11}{3}$

B. $A = 2$, $B = 5$

C. $A = 2$, $B = 1$

D. $A = 1$, $B = 4$

E. $A = \frac{1}{2}$, $B = 4$

2. Suppose that f is a differentiable function on $(0, 4)$, that $f'(x) > 0$ for x in each of the intervals $(0, 1)$, $(1, 2)$ and $(3, 4)$ and that $f'(x) < 0$ on the interval $(2, 3)$.

Select the correct statement.

A. f has a local minimum at 1 and no local maximum.

B. f has a local minimum at 2 and a local maximum at 3.

C. f has a local maximum at 2 and a local minimum at 3.

D. f has local minima at 1 and 3 and a local maximum at 2.

E. f has local maxima at 1 and 3 and a local minimum at 2.

3. $\int_0^{\pi/4} \sin^3 x \cos x \, dx =$ _____

   A. $\frac{1}{4}$
   B. $\frac{3}{50}$
   C. $\frac{1}{6}$
   D. $\frac{66}{1000}$
   E. $\frac{1}{16}$

4. Select the correct statement from below about the function $f(x) = \dfrac{x^2 + 2x - 8}{x - 2}$.

   A. $f(2) = 6$
   B. The function has a jump discontinuity at $x = 2$.
   C. The function is continuous at $x = 2$.
   D. The function has a removable discontinuity at $x = 2$.
   E. The function has an infinite discontinuity (vertical asymptote) at $x = 2$.

5. If $f(x) = \int_0^{5x^2} e^{-t^2}\,dt$, find $f'(x)$.

A. $f'(x) = e^{-x^2}$

B. $f'(x) = 10xe^{-25x^4}$.

C. $f'(x) = 5x^2e^{-x^2}$

D. $f'(x) = e^{-25x^4}$

E. $f'(x) = 5x^2e^{-25x^4}$

6. Find the limit

$$\lim_{x\to 0} \frac{e^x - x - 1}{2 - 2\cos x}.$$

A. 0

B. $-\frac{1}{2}$

C. $\frac{1}{2}$

D. 1

E. Does not exist

7. If $2x + y = 9$, what is the smallest possible value of $4x^2 + 3y^2$?

   A. 60.00

   B. 60.25

   C. 60.50

   D. 60.75

   E. 61.00

8. If $f(1) = 6$, f' is continuous, and $\int_1^8 f'(t)dt = 14$, what is the value of $f(8)$?

   A. 8

   B. 10

   C. 18

   D. 20

   E. 22

9. The linearization for $f(x) = \sqrt{x+3}$ at $x = 1$ is

A. $L(x) = 2 + \frac{1}{4}x - 1$

B. $L(x) = 2 + \frac{1}{4}(x-1)$

C. $L(x) = 2 + \frac{1}{2}(x-1)$

D. $L(x) = 4 + \frac{1}{4}(x-1)$

E. $L(x) = 4 + \frac{1}{8}(x-1)$

10. Find the slope of the tangent line to the curve $x^2 - xy - y^2 = 1$ at the point $(2, -3)$.

A. $-\frac{7}{4}$

B. $-\frac{1}{4}$

C. $\frac{1}{8}$

D. $\frac{1}{4}$

E. $\frac{7}{8}$

11. Find the equation of the tangent line to $g(x) = \dfrac{2x}{1+x^2}$ at $x = 3$.

A. $y = -\dfrac{4}{25}x + \dfrac{3}{25}$

B. $y = \dfrac{1}{3}x - \dfrac{2}{5}$

C. $y = 2x - \dfrac{27}{5}$

D. $y = 16x - \dfrac{474}{10}$

E. $y = -\dfrac{4}{25}x + \dfrac{27}{25}$.

12. Find the derivative of $f(x) = x^2 e^{\cos(2x)}$.

A. $f'(x) = -2(x\sin(2x) - 1)e^{\cos(2x)}$

B. $f'(x) = 2\left(x - x^2\sin(2x)\right)e^{\cos(2x)}$

C. $f'(x) = (x\sin(2x) - 1)e^{\cos(2x)}$

D. $f'(x) = 2xe^{-2\sin(2x)}$

E. $f'(x) = -2x\sin(2x)e^{\cos(2x)}$

13. Given that $f''(x) = 6x - 4$, $f'(1) = 2$, and $f(2) = 10$, find $f(x)$.

A. $f(x) = x^3 - 2x^2 + 10$

B. $f(x) = x^3 - 2x^2 + x + 8$

C. $f(x) = x^3 - 2x^2 + 2x + 6$

D. $f(x) = x^3 - 2x^2 + 3x + 4$

E. $f(x) = x^3 - 2x^2 + 4x + 2$

14. Find $\int_x^{x^2} \sin(2t)\,dt$.

A. $\frac{1}{2}\cos(2x^2) - \frac{1}{2}\cos(2x)$

B. $-\cos(2x^2) + \cos(2x)$

C. $\cos(2x^2 - 2x)$

D. $\cos(2x^2) - \cos(2x)$

E. $-\frac{1}{2}\cos(2x^2) + \frac{1}{2}\cos(2x)$

Free Response Questions
Show all of your work

15. Compute the following general antiderivatives. These are also called indefinite integrals.

(a) $\int \frac{x}{1+x^2}\,dx$

(b) $\int \frac{(\arctan x)^3}{1+x^2}\,dx$

(c) $\int x\sqrt{x-1}\,dx$

16. The tangent line to the graph of a function $f(x)$ at the point $x = 1$ is $y = 5x + 2$.

(a) What is $f(1)$?

(b) What is $f'(1)$?

(c) If $g(x) = f(x^5)$, then find $g'(1)$. Show your work.

17. The velocity of a particle moving on a straight line is

$$v(t) = 3t^2 - 24t + 36 \text{ meters/second},$$

for $0 \leq t \leq 6$.

(a) Find the displacement of the particle over the time interval $0 \leq t \leq 6$. Show your work.

(b) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 6$. Show your work.