MA 113 — Calculus I Fall 2010
Final Exam December 13, 2010

Answer all of the questions 1 - 7 and **two** of the questions 8 - 10. Please indicate which problem is not to be graded by crossing through its number in the table below.

Additional sheets are available if necessary. No books or notes may be used. Please, turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities. Please:

1. clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*,
2. give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Name: ______________________________

Section: __________

Last four digits of student identification number: __________

Question	Score	Total
1		12
2		8
3		12
4		8
5		10
6		10
7		10
8		15
9		15
10		15
Extra Credit		10
Total		100

(1) Compute the following limits:

(a) $\lim_{x \to 1} \frac{\ln x}{\sin(\pi x)}$

(b) $\lim_{x \to \infty} x \sin\left(\frac{\pi}{x}\right)$

(c) $\lim_{t \to \infty} (\sqrt{t^2 + 9} - t)$

(a) $\lim_{x \to 1} \frac{\ln x}{\sin(\pi x)} =$ ____________________________,

(b) $\lim_{x \to \infty} x \sin\left(\frac{\pi}{x}\right) =$ ____________________________,

(c) $\lim_{t \to \infty} (\sqrt{t^2 + 9} - t) =$ ____________________________.

(2) (a) Let $f(x) = x^9 g(2x)$. If $f'(1) = 1$ and $g(2) = -1$, determine $g'(2)$.

(b) If $h(x) = \cos(\ln x - e^{x^2})$, compute $h'(x)$.

(a) $g'(2) =$______________________________,

(b) $h'(x) =$______________________________.

(3) Compute the following integrals:

(a) $\displaystyle\int \left(\sin x - \frac{1}{x^2+1} + 2e^x\right)\, dx$

(b) $\displaystyle\int t\sqrt{t^2+4}\, dt$

(c) $\displaystyle\int_1^x \frac{1 - t\cos t}{t}\, dt$

(a) $\int \left(\sin x - \frac{1}{x^2+1} + 2e^x\right)\, dx =$ ____________________,

(b) $\int t\sqrt{t^2+4}\, dt =$__________________________________,

(c) $\int_1^x \frac{1-t\cos t}{t}\, dt =$__________________________________.

(4) Consider the curve described by the equation $e^{xy} = x - y$.

(a) Find the slope of the tangent line to the curve at the point $(0, -1)$.

(b) Determine the equation of the tangent line to the curve at $(0, -1)$ in the form $y = mx + b$.

(a) The slope is ____________________________________,

(b) The equation of the tangent line is ____________________.

(5) (a) Let $F(x) = \int_1^x \frac{t}{(t+3)^2}\,dt$, where $x > -3$ Determine the interval(s) on which $F(x)$ is increasing.

(b) Let $G(x) = \int_x^3 \left(\frac{1-t}{t}\right)^2 dt$, where $x > 0$. Compute $G''(x)$.

(a) $F(x)$ is increasing on ____________________________,

(b) $G''(x) =$ ___________________________________.

(6) Consider the function $f(x) = \dfrac{1}{x^2}$.

(a) Compute the Riemann sum for $f(x)$ on the interval $[1, 5]$ with $n = 4$ subintervals and by taking the left endpoints as your sample points. (Give your answer as a rational number.)

(b) Determine whether $f(x)$ is increasing or decreasing on $[1, 5]$.

(c) Without computing the integral $\displaystyle\int_1^5 f(x)\,dx$, determine whether the Riemann sum of Part (a) overestimates or underestimates this integral.

(a) The Riemann sum is ______________________________,

(b) $f(x)$ is ____________________________________,

(c) The Riemann sum ______________________ the integral.

(7) Let a and b be positive numbers whose product is 8. Find the minimum value of $a^2 + 2b$. Determine the values of a and b for which the minimum is attained. (As usual, **justify** your answers.)

The minimum value of $a^2 + 2b$ is ____. It is attained if $a =$____ and $b =$____.

Work two of the following three problems. Indicate the problem that is not to be graded by crossing through its number on the front of the exam.

(8) (a) State both parts of the fundamental theorem of calculus. Use complete sentences and be sure to include all assumptions.

(b) Compute the derivative of $F(x) = \int_0^x \sin(\sqrt{t})\,dt$ at $x = \frac{\pi^2}{4}$.

(c) Find the derivative of $G(x) = \int_1^{x^2} (\ln t)^3\,dt$.

(b) $F'(x) =$ ______________________________,

(c) $G'(x) =$ ______________________________.

(9) (a) What does it mean for a function $f(x)$ to be continuous at a point a? Use complete sentences.

(b) Consider the piecewise defined function

$$f(x) = \begin{cases} 2 & \text{if } 0 \leq x \leq 1, \\ ax + 3 & \text{if } 1 < x \leq 2, \\ x^2 - 2x + b & \text{if } 2 \leq x \leq 3, \end{cases}$$

where a and b are constants. Find the values of a and b for which $f(x)$ is continuous on $[0, 3]$.

(b) $a =$ ______________ and $b =$ ______________.

(10) A particle is moving along the x axis with velocity $v(t) = 3t^2 + 3t - 6$ at time t (measured in meters per second).

(a) Find the acceleration $a(t)$ of the particle at time $t = 2$.

(b) Is the particle speeding up or slowing down at time $t = 2$? Justify your answer.

(c) Find all times t at which the particle changes direction. Again, justify your answer.

(d) Find the total distance traveled by the particle during the first 3 seconds.

(a) $a(2) =$ ____________________________,

(b) the particle is ____________________________,

(c) the particle changes direction at ____________,

(d) the total distance traveled is ______________.

Extra Credit Problem:

Check the correct answers below. For each correct answer you earn 2 points, and for each incorrect answer 1 point will be subtracted. Therefore, it might be wise to skip a question rather than risking losing a point. However, your final score on this problem will not be negative! You need not justify your answer.

True	False	
☐	☐	If $f(2) < 0$ and $f(5) > 0$, then there exists a number c between 2 and 5 such that $f(c) = 0$.
☐	☐	If $f'(c) = 0$, then $f(x)$ has a local maximum or minimum at $x = c$.
☐	☐	If f is differentiable function and $f(-2) = f(2)$, then there exists a number c such that $\lvert c\rvert < 2$ and $f'(c) = 0$.
☐	☐	$\int \ln x \, dx = x \ln x - x + C$.
☐	☐	$\int_{-3}^{3} (ax^2 + c)\, dx = 2 \int_{0}^{3} (ax^2 + c)\, dx$ for every choice of a and c.