Worksheet # 2: Functions, Logarithms, and Intro to Limits

1. Let $f(x) = x^3 + 1$ and $g(x) = \sqrt{x}$. Find $(f \circ g)(x)$ and $(g \circ f)(x)$ and specify their domains.

2. Consider the function $f(x) = \sqrt{\dfrac{2}{x^2+3}}$. Find functions $g(x)$ and $h(x)$ so that $f(x)$ can be written as $f(x) = (g \circ h)(x)$.

3. Suppose the graph of $g(x)$ is given by the equation $g(x) = f(2x - 5) + 7$. In terms of standard transformations describe how to obtain the graph of $g(x)$ from the graph of $f(x)$.

4. Find the domain and range of the following functions.

   (a) $f(x) = 15$

   (b) $f(x) = \sqrt{x^2 + 2x + 1}$

5. Compute each of the following logarithms exactly. Do not use a calculator.

   (a) $\log_3(1/27)$

   (b) $\log_2(6) - \log_2(15) + \log_2(20)$

   (c) $\log_{10}(\log_{10}(\log_{10}(10^{10^{100}})))$

6. Solve the following equations for x:

   (a) $10^{2x+1} - 7 = 0$

   (b) $\log_2(x) + \log_2(x - 1) = 1$

7. Sketch the graphs of the following functions using your knowledge of basic functions and transformations. Then sketch the tangent line to the curve at the specified point.

   (a) $f(x) = -|x| + 3$, $x = -1$

   (b) $f(x) = (x - 2)^3 - 1$, $x = 2$

8. A particle is moving along a straight line so that its position at time t seconds is given by $s(t) = 4t^2 - t$.

   (a) Find the average velocity of the particle over the time interval $[1, 2]$.

   (b) Determine the average velocity of the particle over the time interval $[2, t]$ where $t > 2$. Simplify your answer. [Hint: Factor the numerator.]

   (c) Based on your answer in (b) can you guess a value for the instantaneous velocity of the particle at $t = 2$?

9. Let $s(t)$ be the function which describes the position of a particle traveling along the y-axis. Suppose the point $(15, 6)$ is on the graph $y = s(t)$ (in the t-y plane) and the tangent line at this point is given by $y = 6$. At time $t = 15$, determine the particle's position and instantaneous velocity.

10. The point $P(3, 1)$ lies on the curve $y = \sqrt{x - 2}$.

    (a) If Q is the point $(x, \sqrt{x-2})$, find a formula for the slope of the secant line PQ.

    (b) Using your formula from part (a) and a calculator, find the slope of the secant line PQ for the following values of x (do not round until you get to the final answer):

        2.9, 2.99, 2.999, 3.1, 3.01, and 3.001

        TI-8x Calculator Tip: Enter the formula under "y=" and then use "Table".

    (c) Using the results of part (b), guess the value of the slope of the tangent line to the curve at $P(3, 1)$.

    (d) Using the slope from part (c), find the equation of the tangent line to the curve at $P(3, 1)$.

11. True or False:

(a) The graph of every function will pass the vertical line test.

(b) $f \circ g(x) = g \circ f(x)$.

(c) There is a function whose graph is an oval.

(d) $\log_3(3^x) = x$ for all x.