Worksheet # 6: Algebraic Evaluation of Limits, Inverse Functions, and Trigonometric Functions

1. Let $f(x) = \sqrt{x}$

   (a) Let $g(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ and find $g(x)$.

   (b) What is the geometric meaning of $g(4)$?

   (c) What is the domain of $g(x)$?

2. Find the limit or explain why it does not exist. Use the limit rules to justify each step.

   (a) $\lim_{x \to 2} \dfrac{x+2}{x^2-4}$

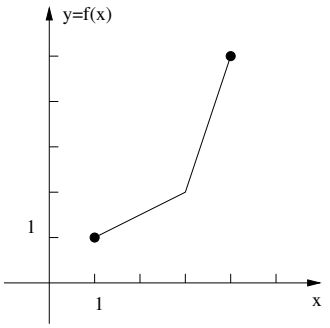
   (b) $\lim_{x \to 2} \dfrac{x-2}{x^2-4}$

   (c) $\lim_{x \to 2} \dfrac{x-2}{x^2+4}$

   (d) $\lim_{x \to 2} (\dfrac{1}{x-2} - \dfrac{3}{x^2-x-2})$.

3. Consider the function $f(x) = 1 + \ln(x)$. Determine the inverse function of f.

4. Consider the function whose graph appears below.



   (a) Find $f(3)$, $f^{-1}(2)$ and $(f(2))^{-1}$.

   (b) Give the domain and range of f and of f^{-1}.

   (c) Sketch the graph of f^{-1}.

5. Convert the angle $\pi/12$ to degrees and the angle 900° to radians. Give exact answers.

6. Find the exact values of the following expressions. Do not use a calculator.

   (a) $\tan^{-1}(1)$

   (b) $\tan(\tan^{-1}(10))$

   (c) $\sin^{-1}(\sin(7\pi/3))$

   (d) $\tan(\sin^{-1}(0.8))$

7. Let O be the center of a circle whose circumference is 48 centimeters. Let P and Q be two points on the circle that are endpoints of an arc that is 6 centimeters long. Find the angle between the segments OQ and OP. Express your answer in radians.

   Find the distance between P and Q.

8. If $\pi/2 \leq \theta \leq 3\pi/2$ and $\tan\theta = 4/3$, find $\sin\theta$, $\cos\theta$, $\cot\theta$, $\sec\theta$, and $\csc\theta$.

9. Find all solutions to the following equations in the interval $[0, 2\pi]$. You will need to use some trigonometric identities.

   (a) $\sqrt{3}\cos(x) + 2\tan(x)\cos^2(x) = 0$

   (b) $3\cot^2(x) = 1$

   (c) $2\cos(x) + \sin(2x) = 0$

10. True or False:

    (a) Let $p(x) = c_n x^n + c_{n-1}x^{n-1} + ... + c_1 x + c_0$ be a polynomial with coefficients $c_n, c_{n-1}, ..., c_0$. Then $\lim_{x\to a} p(x) = c_n a^n + c_{n-1}a^{n-1} + ... + c_1 a + c_0$.

    (b) If $\lim_{x\to 0}(f(x))^2$ exists, then $\lim_{x\to 0} f(x)$ exists.

    (c) Every function has an inverse.

    (d) The graph of every function will pass the horizontal line test.