Worksheet # 10: Derivatives

1. Comprehension check:

   (a) Are differentiable functions also continuous? Are continuous functions also differentiable?

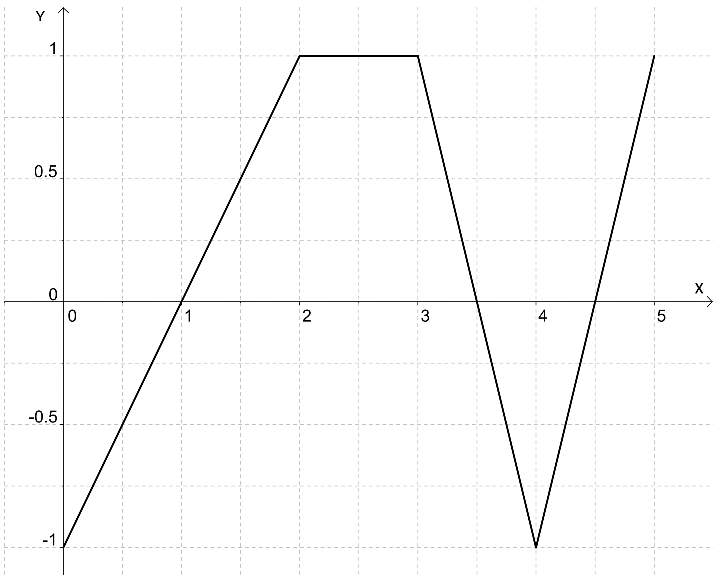
   (b) What does the derivative of $f(x)$ at $x = a$ describe graphically?

   (c) True or false: If $f'(x) = g'(x)$ then $f(x) = g(x)$?

   (d) True or false: $(f(x) + g(x))' = f'(x) + g'(x)$

   (e) How is the number e defined?

2. Consider the graph below of the function $f(x)$ on the interval $[0, 5]$.



   (a) For which x values would the derivative $f'(x)$ not be defined?

   (b) Sketch the graph of the derivative function f'.

3. Find $f'(a)$ using either form of the definition for the derivative:

   (a) $f(x) = 3x^2 - 2x + 1$, $a = 2$.

   (b) $f(x) = \dfrac{1}{x+3}$, $a = -1$.

   (c) $f(x) = \sqrt{x}$, $a = 9$.

4. Let

$$h(t) = \begin{cases} at + b & \text{if } t \leq 0 \\ t^3 + 1 & \text{if } t > 0 \end{cases}.$$

   Find a and b so that h is differentiable at $t = 0$.

5. Compute the derivative of the following functions:

   (a) $f(x) = \dfrac{9}{4}x^8$

   (b) $h(x) = 3e^x + x^2 + 1$

   (c) $k(x) = \dfrac{A}{x^4} + Bx^2 + Cx + D$

   (d) $l(x) = \left(x + \dfrac{1}{x}\right)^2$

6. Find an equation for the tangent line to the curve $y = x^{3/2} + 2$ at $x = 3$.

7. Find the equation of each tangent line to the parabola $y = x^2$ which pass through the point $(0, -1)$. First sketch the graph of the parabola and the desired tangent line(s).

8. Consider the function $f(x) = x^4 - x^3 - 8x^2 + 25x + 10$. Use the Intermediate Value Theorem to show that the graph of f has a horizontal tangent line between $x = -3$ and $x = -2$.

9. Find a function f and a number a so that the following limit represents a derivative $f'(a)$.

$$\lim_{h \to 0} \frac{(4+h)^3 - 64}{h}$$