Worksheet # 15: Related Rates of Change

1. A water tank is shaped like a cone with the vertex pointing down. The height of the tank is 10 feet and diameter of the base 5 feet. At time $t = 0$ the tank is full and starts to be emptied. After 3 minutes the height of the water is 4 feet and it is decreasing at a rate of 1 feet per minute. Find how much water is being pumped out of the tank at this instant. **(Give the units for your answer)**

2. Let a and b denote the lengths of the adjacent and opposite sides of a right angle triangle measured in meters. At time $t = 0$, $a = 20$ and $b = 20$. If a is decreasing at a constant rate of 2 meters per second and b is increasing at a constant rate of 3 meters per second. Find the rate of change of the area of the triangle at time $t = 5$.

3. A car moves along a road that is shaped like the parabola $y = x^2$. At what point on the parabola are the projections to the x and y axis changing at the same rate?

4. A person 6 feet tall walks along a straight path at a rate of 4 feet per second away from a streetlight that is 15 feet above the ground. Find the rate of change of the angle between the person's shadow and the streetlight for any time $t > 0$. Assume that at time $t = 0$ the person is touching the streetlight.

5. The height of a cylinder is a linear function of its radius. The height increases twice as fast as the radius r and $\frac{dr}{dt}$ is constant. At time $t = 1$ seconds the radius is $r = 1$ feet, the height is $h = 3$ feet and the rate of change of the volume is 16π ft/sec. Find the rate of change of the volume when the radius is 4 feet.

6. A car moves at constant speed along a straight road. A house is 3 miles away from the road. When the car is 6 miles away from the house the rate of change of the distance between the car and the house is 52 miles per hour. Find the velocity of the car.

7. Let $f(x) = \dfrac{1}{1 + \frac{1}{x}}$ and $h(x) = \dfrac{1}{1 + \frac{1}{f(x)}}$

   (a) Find $f'(x)$.

   (b) Use the previous result to find $h'(x)$.

   (c) Let $x = x(t)$ be a function of time t if $x(1) = 1$, find $\dfrac{dx}{dt}$ given that

   $$\frac{d}{dt}h(x(1)) = 18$$

8. An spherical snow ball is melting. The rate of change of the surface area of the snow ball is constant and equal to 1 inch per hour. Find the rate of change of the radius of the snow ball when $r = 5$ inches.