Worksheet # 16: Review for Exam II

1. Evaluate the following limits;

   (a) $\lim_{x\to\infty} \dfrac{7x^8 + 3x^3 - 1}{21x^3 - 13x^8 + x^2}$

   (b) $\lim_{x\to\infty} \dfrac{2012 - x^{2012}}{2013 - x^{2013}}$

   (c) $\lim_{x\to-\infty} \dfrac{2012 - x^{2012}}{2013 - x^{2013}}$

   (d) $\lim_{x\to-\infty} \dfrac{5x}{\sqrt{9x^2 + 1}}$

2. If $g(x) = x^2 + 5^x - 3$, use the Intermediate Value Theorem to show that there is a number a such that $g(a) = 10$.

3. (a) State the definition of the derivative of a function $f(x)$ at a point a.

   (b) Find a function f and a number a such that

   $$\frac{f(x) - f(a)}{x - a} = \frac{\ln(2x-1)}{x-1}$$

   (c) Evaluate the following limit by using (a) and (b),

   $$\lim_{x\to 1} \frac{\ln(2x-1)}{x-1}$$

4. State the following rules with the hypotheses and conclusion.

   (a) The product rule and quotient rule.

   (b) The chain rule.

5. A particle is moving along a line so that at time t seconds, the particle is $s(t) = \frac{1}{3}t^3 - t^2 - 8t$ meters to the right of the origin.

   (a) Find the time interval(s) when the particle is moving to the left.

   (b) Find the time(s) when the velocity is zero.

   (c) Find the time interval(s) when the particle's velocity is increasing.

   (d) Find the time interval(s) when the particle is speeding up.

6. Compute the first derivative of each of the following functions:

   (a) $f(x) = \cos(4\pi x^3) + \sin(3x + 2)$

   (b) $b(x) = x^4 \cos(3x^2)$

   (c) $y(\theta) = e^{\sec(2\theta)}$

   (d) $k(x) = \ln(7x^2 + \sin(x) + 1)$

   (e) $u(x) = (\sin^{-1}(2x))^2$

   (f) $h(x) = \dfrac{8x^2 - 7x + 3}{\cos(2x)}$

   (g) $m(x) = \sqrt{x} + \dfrac{1}{\sqrt[3]{x^4}}$

   (h) $q(x) = \dfrac{e^x}{1 + x^2}$

   (i) $n(x) = \cos(\tan(x))$

   (j) $w(x) = \arcsin(x) \cdot \arccos(x)$

7. Let $f(x) = \cos(2x)$. Find the fourth derivative at $x = 0$, $f^{(4)}(0)$.

8. Let f be a one to one, differentiable function such that $f(1) = 2,\ f(2) = 3,\ f'(1) = 4$ and $f'(2) = 5$. Find the derivative of the inverse function, $(f^{-1})'(2)$.

9. The tangent line to $f(x)$ at $x = 3$ is given by $y = 2x - 4$. Find the tangent line to $g(x) = \dfrac{x}{f(x)}$ at $x = 3$. Put your answer in slope-intercept form.

10. Consider the curve $xy^3 + 12x^2 + y^2 = 24$. Assume this equation can be used to define y as a function of x (i.e. $y = y(x)$) near $(1, 2)$ with $y(1) = 2$. Find the equation of the tangent line to this curve at $(1, 2)$.

11. Let x be the angle in the interval $\left(\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right)$ so that $\sin(x) = -\dfrac{3}{5}$. Find: $\sin(-x), \cos(x)$, and $\cot(x)$.

12. Each side of a square is increasing at a rate of 5 cm/s. At what rate is the area of the square increasing when the area is 14 cm^2?

13. The sides of a rectangle are varying in such a way that the area is constant. At a certain instant the length of a rectangle is 16 m, the width is 12 m and the width is increasing at 3 m/s. What is the rate of change of the length at this instant?

14. Suppose f and g are differentiable functions such that $f(2) = 3$, $f'(2) = -1$, $g(2) = \frac{1}{4}$, and $g'(2) = 2$. Find:

    (a) $h'(2)$ where $h(x) = \ln([f(x)]^2)$;

    (b) $l'(2)$ where $l(x) = f(x^3 \cdot g(x))$.

15. Abby is driving north along Ash Road. Boris driving west on Birch Road. At 11:57 am, Boris is 5 km east of Oakville and traveling west at a speed of 60 km/h and Abby is 10 km north of Oakville and traveling north at a speed of 50 km/h.

    (a) Make a sketch showing the location and direction of travel for Abby and Boris.

    (b) Find the rate of change of the distance between Abby and Boris at 11:57 AM.

    (c) At 11:57 AM, is the distance between Abby and Boris increasing, decreasing, or not changing?