

## Worksheet # 22: Newton's Method and Antiderivatives

1. Use Newton's method to find an approximation to  $\sqrt[3]{2}$ . You may do this by finding a solution of  $x^3 - 2 = 0$ .
2. Use Newton's method to approximate the critical points of the function  $f(x) = x^5 - 7x^2 + x$ .
3. Let  $f(x) = \frac{x}{1+x^2}$ .
  - (a) Solve  $f(x) = 0$  without using Newton's method.
  - (b) Use Newton's method to solve  $f(x) = 0$  beginning with the starting point  $x_1 = 2$ . Does something interesting happen?
  - (c) Make a sketch of the graph of  $f$  and explain what you observed in part b).

4. (a) Let  $f(x) = \frac{x^3}{3} + 1$ . Calculate the derivative  $f'(x)$ . What is an anti-derivative of  $f'(x)$ ?  
(b) Let  $g(x) = x^2 + 1$ . Let  $G(x)$  be any anti-derivative of  $g$ . What is  $G'(x)$ ?

5. Find  $f$  given that

$$f'(x) = \sin(x) - \sec(x) \tan(x), \quad f(\pi) = 1.$$

6. Find  $g$  given that

$$g''(t) = -9.8, \quad g'(0) = 1, \quad g(0) = 2.$$

On the surface of the earth, the acceleration of an object due to gravity is approximately  $-9.8 \text{ m/s}^2$ . What situation could we describe using the function  $g$ ? Be sure to specify what  $g$  and its first two derivatives represent.

7. A small rock is dropped from a bridge and the splash is heard 3 seconds later. How high is the bridge?
8. Let  $f$  be a function on the domain  $(-\infty, \infty)$  that satisfies  $(f')^2 = 1$ . This is an example of a *differential equation*. Suppose also that we are given an *initial value condition*  $f(0) = 1$ .
  - (a) Show that this does not have a unique solution by finding two different functions that satisfy both conditions.
  - (b) What does the fact that there are multiple solutions say about this as a model for physical phenomena?
9. Find a function  $f(x)$  such that  $f'(x) = f(x)$ . Find the solution, given initial condition  $f(0) = \pi$ .
10. Let  $f(x) = 1/x$ ,  $F(x) = \ln(|x|)$ , and

$$G(x) = \begin{cases} \ln(x), & x > 0 \\ \ln(-x) + 8, & x < 0. \end{cases}$$

- (a) Is  $F$  an anti-derivative of  $f$ ? Is  $G$  an anti-derivative of  $f$ ? Is  $F - G$  equal to a constant?
- (b) Does Theorem 1 on page 275 imply that  $F - G$  is constant? Is the theorem wrong?