Worksheet # 22 and 23: Newton's Method, Antiderivatives, and Area

Due to election day on Tuesday, we have combined two worksheets into one and deleted about half the problems. The full worksheets are available at `http://www.math.uky.edu/~ma113/`

1. Use Newton's method to find an approximation to $\sqrt[3]{2}$. You may do this by finding a solution of $x^3 - 2 = 0$.

2. Use Newton's method to approximate the critical points of the function $f(x) = x^5 - 7x^2 + x$.

3. (a) Let $f(x) = \dfrac{x^3}{3} + 1$. Calculate the derivative $f'(x)$. What is an anti-derivative of $f'(x)$?

   (b) Let $g(x) = x^2 + 1$. Let $G(x)$ be any anti-derivative of g. What is $G'(x)$?

4. Find f given that

$$f'(x) = \sin(x) - \sec(x)\tan(x)$$
$$f(\pi) = 1.$$

5. Find g given that

$$g''(t) = -9.8, \qquad g'(0) = 1, \qquad g(0) = 2.$$

   On the surface of the earth, the acceleration of an object due to gravity is approximately $-9.8\,\mathrm{m/s^2}$. What situation could we describe using the functino g? Be sure to specify what g and its first two derivatives represent.

6. Write each of following in summation notation:

   (a) $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$

   (b) $2 + 4 + 6 + 8 + 10 + 12 + 14$

   (c) $2 + 4 + 8 + 16 + 32 + 64 + 128$.

7. Compute $\displaystyle\sum_{i=1}^{4}\left(\sum_{j=1}^{3}(i+j)\right)$.

   The following formulae will be useful below.

$$\sum_{j=1}^{n} j = \frac{n(n+1)}{2}, \qquad \sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}$$

8. Find the number n such that $\sum_{i=1}^{n} i = 78$.

9. Give the value of the following sums.

   (a) $\displaystyle\sum_{j=1}^{20}(2k^2 + 3)$

   (b) $\displaystyle\sum_{j=11}^{20}(3k + 2)$

10. Let $f(x) = \sqrt{1 - x^2}$. Divide the interval $[0, 1]$ into four equal subintervals and compute L_4 and R_4, the left and right-endpoint approximations to the area under the graph of f. Is R_4 larger or smaller than the true area? Is L_4 larger or smaller than the true area? What can you conclude about the value π?

11. Let $f(x) = x^2$.

    (a) If we divide the interval $[0, 2]$ into n equal intervals of equal length, how long is each interval?

    (b) Write a sum which gives the right-endpoint approximation R_n to the the area under the graph of f on $[0, 2]$.

    (c) Use one of the formulae for the sums of powers of k to find a closed form expression for R_n.

    (d) Take the limit of R_n as n tends to infinity to find an exact value for the area.