Worksheet # 28: Review I for Final

1. Compute the derivative of the given function:

   (a) $f(\theta) = \cos(2\theta^2 + \theta + 2)$

   (b) $g(u) = \ln\left(\sin^2(u)\right)$

   (c) $h(x) = \int_{-3599}^{x} (t^2 - te^{t^2+t+1})\, dt$

   (d) $r(y) = \arccos(y^3 + 1)$

2. Compute the following definite integrals:

   (a) $\int_0^{\pi} \sec^2(t/4)\, dt$

   (b) $\int_0^1 xe^{-x^2}\, dx$

   (c) $\int_0^1 x(1+x)^6\, dx$

   (d) $\int_0^{\pi/4} \sin(2x)\cos(2x)\, dx$

3. What is the area of the bounded region bounded by $f(x) = \frac{1}{x}$, $x = e^2$, $x = e^8$ and x-axis? Sketching the region might be helpful.

4. If $F(x) = \int_{3x^2+1}^{7} \cos(t^2)\, dt$, find $F'(x)$. Justify your work.

5. Suppose a bacteria colony grows at a rate of $r(t) = 100\, e^{0.02t}$ with t given in hours. What is the growth in population from time $t = 1$ to $t = 3$?

6. Use the left endpoint approximation with 4 equal subintervals to estimate the value of $\int_1^5 x^2\, dx$. Will this estimate be larger or smaller than the actual value of definite integral? Explain your answer.

7. Find an antiderivative for the function $f(x) = \dfrac{1+x}{1+x^2}$.

8. Give the interval(s) for which the function F is increasing. The function F is defined by

$$F(x) = \int_0^x \frac{5t-3}{t^2+10}\, dt$$

9. Find a function $f(x)$ such that $f(e) = 0$ and $f'(x) = \dfrac{e^x - e^{-x}}{\ln x}$. (Hint: Consider the Fundamental Theorem of Calculus)

10. Which of the following is an antiderivative for the function $f(x) = 2x\cos(x^2)$. Circle all the correct answers.

    (a) $F(x) = -\sin(x^2)$

    (b) $F(x) = \sin(x^2)$

    (c) $F(x) = \int_0^{x^2} \sin(t)\, dt$

    (d) $F(x) = \int_0^x 2t\cos(t^2)\, dt$

    (e) $F(x) = \int_0^x 2t\sin(t^2)\, dt$

    (f) $F(x) = \int_0^{x^2} \cos(t)\, dt$

11. Which of the following integrals are the same as $\int_{1/2}^{1} \dfrac{\ln(\arcsin(x))}{\arcsin(x)\sqrt{1-x^2}}\, dx$. Circle all the correct answers.
    (Hint: Use substitution method. You may need to do substitution more than once.)

(a) $\displaystyle\int_{1/2}^{1} \frac{\ln(u)}{u}\,du$

(b) $\displaystyle\int_{\pi/6}^{\pi/2} \frac{\ln(u)}{u}\,du$

(c) $\displaystyle\int_{1/2}^{1} t\,dt$

(d) $\displaystyle\int_{\ln(\pi/6)}^{\ln(\pi/2)} t\,dt$

(e) $\displaystyle\int_{\pi/6}^{\pi/2} t\,dt$

12. Compute the indefinite integral $\int \dfrac{\sin(x)}{1+\cos^2(x)}\,dx$.

13. Let $F(x) = \displaystyle\int_0^x \sin^2(t)\,dt$. Evaluate the limit

$$\lim_{x\to 0} \frac{F(x)}{x^2}.$$

14. Evaluate $\dfrac{d}{dx}(x^5 \displaystyle\int_2^{x^5} \frac{\sin(t)}{t}\,dt)$.