## Exam 1

Name: $\qquad$ Section and/or TA: $\qquad$
Do not remove this answer page - you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 10 multiple choice questions and 6 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems. The wise student will show work for the multiple choice problems as well.

## Multiple Choice Questions

1 (A) B C D E
2 (A) B (C) D (E)
6 (A B C D E
7 (A) B C D E
3 (A) B C D E
8 (A B C D E
4 (A) B (C) D (E)
9 (A) B C D E
5 (A) B (C) D E
10 (A) B (C) D E

## SCORE

| Multiple <br> Choice | 11 | 12 | 13 | 14 | 15 | 16 | Total <br> Score |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 50 | 9 | 6 | 13 | 12 | 5 | 5 | 100 |
|  |  |  |  |  |  |  |  |

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## Multiple Choice Questions

1. Find $\int \sin (x) \cos (\cos (x)) d x$
A. $\cos (\sin (x))+C$
B. $-\sin (\cos (x))+C$
C. $-\cos (x) \sin (\sin (x))+C$
D. $\sin (x)+\cos (x)+C$
E. $\sin ^{2}(x)+\cos ^{2}(x)+C$
2. $\int x \cos (2 x) d x=$.
A. $-\frac{x}{2} \sin (2 x)+\frac{1}{4} \cos (2 x)+C$
B. $\frac{x}{2} \sin (2 x)-\frac{1}{4} \cos (2 x)+C$
C. $\frac{x}{2} \sin (2 x)+\frac{1}{4} \cos (2 x)+C$
D. $-2 x \sin (2 x)+\cos (2 x)+C$
E. $-2 x \sin (2 x)-4 \cos (2 x)+C$
3. Which of the following is the correct form of the partial fraction expansion of

$$
\frac{x^{2}+3 x-10}{\left(x^{2}+4 x+6\right)\left(x^{2}-1\right)(x+1)} ?
$$

A. $\frac{A}{x-1}+\frac{B}{(x-1)^{2}}+\frac{C}{x+1}+\frac{D x+E}{x^{2}+4 x+6}$.
B. $\frac{A}{x+1}+\frac{B}{(x+1)^{2}}+\frac{C}{x-1}+\frac{D x+E}{x^{2}+4 x+6}$.
C. $\frac{B}{(x+1)^{2}}+\frac{C}{x-1}+\frac{D x+E}{x^{2}+4 x+6}$.
D. $\frac{A}{x+1}+\frac{B x+C}{x^{2}-1}+\frac{D x+E}{x^{2}+4 x+6}$.
E. $\frac{A}{x+1}+\frac{B}{(x+1)^{2}}+\frac{C}{x-1}+\frac{D}{x^{2}+4 x+6}$.
4. Use the fact that

$$
\frac{13 x^{2}+6 x-24}{(3 x-1)\left(x^{2}+4\right)}=\frac{6 x+4}{x^{2}+4}-\frac{5}{3 x-1}
$$

to evaluate the integral

$$
\int \frac{13 x^{2}+6 x-24}{(3 x-1)\left(x^{2}+4\right)} d x
$$

A. $6 \ln \left|x^{2}+4\right|-\frac{5}{3} \ln |3 x-1|+C$
B. $12 \arctan \left(\frac{x}{2}\right)-\frac{5}{3} \ln |3 x-1|+C$
C. $\frac{8}{x+2}-\frac{5}{3} \ln |3 x-1|+6 \ln |x+2|+C$
D. $3 \ln \left|x^{2}+4\right|-\frac{5}{3} \ln |3 x-1|+2 \arctan \left(\frac{x}{2}\right)+C$
E. $4 \ln |x-2|+2 \ln |x+2|-\frac{5}{3} \ln |3 x-1|+C$
5. What is the area of the region enclosed by the curves $y^{2}=x$ and $y=x$ ?
A. $1 / 2$
B. $1 / 3$
C. 1
D. $1 / 6$
E. $3 / 2$
6. The table below gives values of $f, f^{\prime}, g$, and $g^{\prime}$ for selected values of $x$. If

$$
\int_{0}^{1} f^{\prime}(x) g(x) d x=5
$$

then $\int_{0}^{1} f(x) g^{\prime}(x) d x=$

| $x$ | 0 | 1 |
| :---: | :---: | :---: |
| $f(x)$ | 2 | 4 |
| $f^{\prime}(x)$ | 6 | -3 |
| $g(x)$ | -4 | 3 |
| $g^{\prime}(x)$ | 2 | -1 |

A. -14
B. -13
C. -2
D. 7
E. 15
7. Which trigonometric integral is obtained after trigonometric substitution for

$$
\int \frac{\sqrt{4-x^{2}}}{x} d x
$$

A. $\int \tan (\theta) d \theta$
B. $\frac{1}{2} \int \frac{\sin ^{2}(\theta)}{\cos (\theta)} d \theta$
C. $2 \int \frac{\cos ^{2}(\theta)}{\sin (\theta)} d \theta$
D. $\int \sin (\theta) d \theta$
E. $\int \cos (\theta) d \theta$
8. The graph of the function $f$ is shown below for $0 \leq x \leq 3$. Of the following, which has the smallest value?

A. $\int_{1}^{3} f(x) d x$
B. Left Riemann sum approximation of $\int_{1}^{3} f(x) d x$ with 6 subintervals of equal length.
C. Right Riemann sum approximation of $\int_{1}^{3} f(x) d x$ with 6 subintervals of equal length.
D. Midpoint sum approximation of $\int_{1}^{3} f(x) d x$ with 6 subintervals of equal length.
E. Trapezoidal sum approximation of $\int_{1}^{3} f(x) d x$ with 6 subintervals of equal length.
9. The function $f$ is continuous on the closed interval $[2,14]$ and has values as shown the table below. Using three subintervals, what is the approximation of $\int_{2}^{14} f(x) d x$ found by using the Trapezoid rule?

| $x$ | 2 | 6 | 10 | 14 |
| :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 12 | 28 | 34 | 30 |

A. 249
B. 296
C. 332
D. 368
E. $387 . \overline{33}$
10. What are all the values of $p$ for which $\int_{1}^{\infty} \frac{1}{x^{2 p}} d x$ converges
A. $p<-1$
B. $p>0$
C. $p>\frac{1}{2}$
D. $p>1$
E. There are no values of $p$ for which this integral converges.

## Free Response Questions

11. Find the following definite integral

$$
\int_{0}^{\sqrt[3]{\pi / 2}} v^{2} \cos \left(v^{3}\right) d v
$$

12. Consider the two curves $y=x^{2}+2$ and $y=x+1$.
(a) Find the area of the region enclosed by these two curves and the vertical lines $x=0$ and $x=3$.
(b) There is a vertical line $x=a$ for $0<a<3$ so that the area between the two curves between $x=0$ and $x=a$ is exactly $\frac{3}{2}$. Write down the cubic equation that you need to solve in order to find $a$.
13. Find the following antiderivatives:
(a) $\int x^{2} \sin (x) d x$
(b) $\int x^{5} \sqrt[3]{x^{3}+1} d x$
14. Find the following integrals
(a) $\int_{1}^{\infty} \frac{x}{\left(1+x^{2}\right)^{2}} d x$.
(b) $\int_{0}^{8} x^{-2 / 3} d x$.
15. Compute the integral

$$
\int \sin ^{3}(x) d x
$$

Hint: Note that $\sin ^{3}(x)=\sin ^{2}(x) \sin (x)$ and use a trigonometric identity followed by substitution.
16. A table of values for a continuous function $f$ is shown below. If four equal subintervals of $[0,2]$ are used, what is the Simpson's rule approximation for $\int_{0}^{2} f(x) d x$.

| $x$ | 0.0 | 0.5 | 1.0 | 1.5 | 2.0 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 2 | 8 | 6 | 12 | 10 |

