MA 114 - Calculus II Exam 4

Spring 2015
May. 4, 2015

Name: $\qquad$

Section: $\qquad$

Last 4 digits of student ID \#: $\qquad$

- No books or notes may be used.
- Turn off all your electronic devices and do not wear ear-plugs during the exam.
- You may use a calculator, but not one which has symbolic manipulation capabilities or a QWERTY keyboard.
- Additional blank sheets for scratch work are available upon request.
- Multiple Choice Questions:

Record your answers on the right of this cover page by marking the box corresponding to the correct answer.

- Free Response Questions:

Show all your work on the page of the problem. Clearly indicate your answer and the reasoning used to arrive at that answer.

Multiple Choice Answers

| Question |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | A | B | C | D | E |
| 2 | A | B | C | D | E |
| 3 | A | B | C | D | E |
| 4 | A | B | C | D | E |
| 5 | A | B | C | D | E |
| 6 | A | B | C | D | E |
| 7 | A | B | C | D | E |

Exam Scores

| Question | Score | Total |
| :---: | ---: | ---: |
| MC |  | 28 |
| 8 |  | 15 |
| 9 |  | 15 |
| 10 |  | 10 |
| 11 |  | 12 |
| 12 |  | 20 |
| Total |  | 100 |

Unsupported answers for the free response questions may not receive credit!

Feel free to use the following identities:

$$
\sin ^{2} \theta=\frac{1-\cos (2 \theta)}{2}, \quad \cos ^{2} \theta=\frac{1+\cos (2 \theta)}{2}, \quad \sin (2 \theta)=2 \sin \theta \cos \theta
$$

Record the correct answer to the following problems on the front page of this exam.

1. Find the antiderivative of $(x+1) \sin (2 x)$ :
A. $\cos (2 x)+\frac{(x+1)^{2}}{2} \sin (2 x)+C$
B. $\frac{-(x+1)}{2} \cos (2 x)+\frac{\sin (2 x)}{4}+C$
C. $(x+1) \sin (2 x)-\cos (2 x)+C$
D. $\sin (2 x)-(x+1) \cos (2 x)+C$
E. $\frac{x+1}{2} \cos (2 x)-\frac{1}{4} \sin (2 x)+C$
2. The rectangular coordinates for the point with polar coordinates $r=3$ and $\theta=2 \pi / 3$ are
A. $\left(3, \frac{2 \pi}{3}\right)$
B. $\left(-\frac{2 \pi}{3}, 3\right)$
C. $\left(\frac{-3}{2}, \frac{3 \sqrt{3}}{2}\right)$
D. $\left(\sqrt{9+\frac{4 \pi^{2}}{9}}, \arctan \left(\frac{2 \pi}{9}\right)\right)$
E. $(0,-3)$
3. Which of the following improper integrals converge?
I. $\int_{1}^{\infty} \frac{d x}{x^{2}}$,
II. $\int_{0}^{1} \frac{d x}{x^{3}}$,
III. $\int_{1}^{\infty} \frac{d x}{(x-2)^{4}}$
A. I. only
B. II. only
C. I. and III. only
D. all of these converge
E. none of these converge
4. Which of the following is true for the infinite series $\sum_{n=1}^{\infty} \frac{(-1)^{n}}{n^{3}+1}$ ?
A. It is divergent.
B. It is absolutely convergent, but divergent.
C. It is convergent, but not absolutely convergent.
D. It is absolutely convergent and convergent.
E. None of the above.
5. The power series

$$
\sum_{n=1}^{\infty}(-1)^{n+1} \frac{x^{2 n}}{n}
$$

is a Taylor series for which function?
A. $\sin (-x)$
B. $\arctan (-x)$
C. $\ln (1-x)$
D. $x e^{-x}$
E. $\ln \left(1+x^{2}\right)$
6. What is the area of the surface obtained by rotating the curve $y=\frac{x^{3}}{3}$, for $0 \leq x \leq 1$, around the $x$-axis?
A. $\frac{\pi}{81}(10 \sqrt{10}-1)$
B. $\frac{4 \pi}{45}(1+\sqrt{2})$
C. $\frac{\pi}{9}(2 \sqrt{2}-1)$
D. $\frac{110}{101}$
E. 6.8451

Record the correct answer to the following problems on the front page of this exam.
7. Which of the following describes the behavior of the solution to the differential equation $y^{\prime}=3(y-10)$ with initial condition $y(0)=5$ ?
A. $\lim _{t \rightarrow \infty} y(t)=3$
B. $\lim _{t \rightarrow \infty} y(t)=5$
C. $\lim _{t \rightarrow \infty} y(t)=10$
D. $\lim _{t \rightarrow \infty} y(t)=\infty$
E. $\lim _{t \rightarrow \infty} y(t)=-\infty$
8. Consider the curve parametrized by

$$
x(t)=t^{2}-1, \quad y(t)=t^{3}-3 t .
$$

(a) Find the value(s) of $t$ that correspond to the point $(2,0)$ under this parametrization.
(b) Find the equation for the tangent line(s) to the curve at the point $(2,0)$.
(c) Find the point(s) where the tangent line to the curve is vertical.
9. (a) Find the general solution of the differential equation

$$
y^{\prime}+x y=(x+1) e^{x} .
$$

(b) Solve the initial value problem

$$
y^{\prime}+4 x y^{2}=0, \quad y(1)=-1
$$

10. Consider the curve parametrized by

$$
x(t)=2 t^{2}+6 t+5, \quad y(t)=t^{2}+3 t, \quad 0 \leq t \leq 2 .
$$

(a) Find the speed of the parametrization at time $t$.
(b) Find a value for $t$ where the speed is equal to 10 .
(c) Find the length of this curve.
11. Consider the power series

$$
\sum_{n=1}^{\infty} \frac{(-1)^{n} n^{2}}{3^{n}}(x-5)^{n}
$$

(a) Determine the radius of convergence of the power series above. Clearly indicate which tests you use, and verify that all necessary assumptions are satisfied.
(b) Determine the interval of convergence of the power series above. Clearly indicate which tests you use, and verify that all necessary assumptions are satisfied.
12. (a) Let $R_{1}$ be the region in the first quadrant bounded above by $f(x)=\frac{1}{1+x^{2}}$, on the sides by $x=0$ and $x=1$, and below by the $x$-axis.

Use the Disk/Washer method to find the volume of the solid obtained by rotating $R_{1}$ around the $x$-axis. Give an exact value, not a decimal approximation.
(b) Let $R_{2}$ be the region in the first quadrant bounded above by $f(x)=\frac{1}{(x+1)(x+2)}$, on the sides by $x=1$ and $x=3$, and below by the $x$-axis.

Use the Shell method to find the volume of the solid obtained by rotating $R_{2}$ around the $y$-axis. Give an exact value, not a decimal approximation.

