MA 114 Worksheet #17: Average value of a function

1. Write down the equation for the average value of an integrable function $f(x)$ on $[a, b]$.

2. Find the average value of the following functions over the given interval.

   (a) $f(x) = x^3,\ [0, 4]$

   (b) $f(x) = x^3,\ [-1, 1]$

   (c) $f(x) = \cos(x),\ \left[0, \frac{\pi}{6}\right]$

   (d) $f(x) = \dfrac{1}{x^2+1},\ [-1, 1]$

   (e) $f(x) = \dfrac{\sin(\pi/x)}{x^2},\ [1, 2]$

   (f) $f(x) = e^{-nx},\ [-1, 1]$

   (g) $f(x) = 2x^3 - 6x^2,\ [-1, 3]$

   (h) $f(x) = x^n$ for $n \geq 0$, $[0, 1]$

3. In a certain city the temperature (in $^\circ F$) t hours after 9 am was modeled by the function $T(t) = 50 + 14 \sin \dfrac{\pi t}{12}$. Find the average temperature during the period from 9 am to 9 pm.

4. The velocity v of blood that flows in a blood vessel with radius R and length l at a distance r from the central axis is

$$v(r) = \frac{P}{4\eta l}(R^2 - r^2)$$

   where P is the pressure difference between the ends of the vessel and η is the viscosity of the blood. Find the average velocity (with respect to r) over the interval $0 < r < R$. Compare the average velocity with the maximum velocity.

5. Breathing is cyclic and a full respiratory cycle from the beginning of inhalation to the end of exhalation takes about 5 s. The maximum rate of air flow into the lungs is about 0.5 L/s. This explains, in part, why the function $f(t) = \frac{1}{2}\sin(2\pi t/5)$ has often been used to model the rate of air flow into the lungs. Use this model to find the volume of inhaled air in the lungs at time t. Then use this new function to compute the average volume of inhaled air in the lungs in one respiratory cycle.

MA 114 Worksheet #18: Volumes I

1. If a solid has a cross-sectional area given by the function $A(x)$, what integral should be evaluated to find the volume of the solid?

2. Calculate the volume of the following solid. The base is a square, one of whose sides is the interval $[0, l]$ along the x-axis. The cross sections perpendicular to the x-axis are rectangles of height $f(x) = x^2$.

3. Calculate the volume of the following solid. The base is the region enclosed by $y = x^2$ and $y = 3$. The cross sections perpendicular to the y-axis are squares.

4. The base of a certain solid is the triangle with vertices at $(-10, 5)$, $(5, 5)$, and the origin. Cross-sections perpendicular to the y-axis are squares. Find the volume of the solid.

5. Calculate the volume of the following solid. The base is a circle of radius r centered at the origin. The cross sections perpendicular to the x-axis are squares.

6. Calculate the volume of the following solid. The base is the parabolic region $\{(x, y) \mid x^2 \leq y \leq 4\}$. The cross sections perpendicular to the y-axis are right isosceles triangles whose hypotenuse lies in the region.

7. Sketch the solid given by the integral
$$\pi \int_0^1 (y^2 + 1)^2 - 1 \, dy.$$

8. For each of the following, use disks or washers to find the an integral expression for the volume of the region. Evaluate the integrals for parts (a) and (d).
   (a) R is region bounded by $y = 1 - x^2$ and $y = 0$; about the x-axis.
   (b) R is region bounded by $y = \dfrac{1}{x}$, $x = 1$, $x = 2$, and $y = 0$; about the x-axis.
   (c) R is region bounded by $x = 2\sqrt{y}$, $x = 0$, and $y = 9$; about the y-axis.
   (d) R is region bounded by $y = 1 - x^2$ and $y = 0$; about the line $y = -1$.
   (e) Between the regions in part (a) and part (d), which volume is bigger? Why?
   (f) R is region bounded by $y = e^{-x}$, $y = 1$, and $x = 2$; about the line $y = 2$.
   (g) R is region bounded by $y = x$ and $y = \sqrt{x}$; about the line $x = 2$.

9. Find the volume of the cone obtained by rotating the region under the segment joining $(0, h)$ and $(r, 0)$ about the y-axis.

10. The torus is the solid obtained by rotating the circle $(x - a)^2 + y^2 = b^2$ around the y-axis (assume that $a > b$). Show that it has volume $2\pi^2 ab^2$.
[Hint: Draw a picture, set up the problem and evaluate the integral by interpreting it as the area of a circle.]

MA 114 Worksheet #19: Volumes II

1. (a) Write a general integral to compute the volume of a solid obtained by rotating the region under $y = f(x)$ over the interval $[a, b]$ about the y-axis using the method of cylindrical shells.

   (b) If you use the disk method to compute the same volume, are you integrating with respect to x or y? Why?

2. Sketch the enclosed region and use the Shell Method to calculate the volume of rotation about the y-axis.

   (a) $y = 3x - 2$, $y = 6 - x$, $x = 0$

   (b) $y = x^2$, $y = 8 - x^2$, $x = 0$, for $x \geq 0$

   (c) $y = 8 - x^3$, $y = 8 - 4x$, for $x \geq 0$

3. For each of the following, use disks or washers to find the an integral expression for the volume of the region. Evaluate the integrals for parts (a) and (d).

   (a) R is region bounded by $y = 1 - x^2$ and $y = 0$; about the x-axis.

   (b) R is region bounded by $y = \dfrac{1}{x}$, $x = 1$, $x = 2$, and $y = 0$; about the x-axis.

   (c) R is region bounded by $x = 2\sqrt{y}$, $x = 0$, and $y = 9$; about the y-axis.

   (d) R is region bounded by $y = 1 - x^2$ and $y = 0$; about the line $y = -1$.

   (e) Between the regions in part (a) and part (d), which volume is bigger? Why?

   (f) R is region bounded by $y = e^{-x}$, $y = 1$, and $x = 2$; about the line $y = 2$.

   (g) R is region bounded by $y = x$ and $y = \sqrt{x}$; about the line $x = 2$.

4. A soda glass has the shape of the surface generated by revolving the graph of $y = 6x^2$ for $0 \leq x \leq 1$ about the y-axis. Soda is extracted from the glass through a straw at the rate of $1/2$ cubic inch per second. How fast is the soda level in the glass dropping when the level is 2 inches? (Answer should be implicitly in units of inches per second.)

5. The torus is the solid obtained by rotating the circle $(x-a)^2 + y^2 = b^2$ around the y-axis (assume that $a > b$). Show that it has volume $2\pi^2ab^2$.
   [Hint: Draw a picture, set up the problem and evaluate the integral by interpreting it as the area of a circle.]

MA 114 Worksheet #20: Arc length and surface area

1. (a) Write down the formula for the arc length of a function $f(x)$ over the interval $[a, b]$ including the required conditions on $f(x)$.
   (b) Write down the formula for the surface area of a solid of revolution generated by rotating a function $f(x)$ over the interval $[a, b]$ around the x-axis. Include the required conditions on $f(x)$.
   (c) Write down the formula for the surface area of a solid of revolution generated by rotating a function $f(x)$ over the interval $[a, b]$ around the y-axis. Include the required conditions on $f(x)$.

2. Find an integral expression for the arc length of the following curves. Do ***not*** evaluate the integrals.
   (a) $f(x) = \sin(x)$ from $x = 0$ to $x = 2$.
   (b) $f(x) = x^4$ from $x = 2$ to $x = 6$.
   (c) $x^2 + y^2 = 1$

3. Find the arc length of the following curves.
   (a) $f(x) = x^{3/2}$ from $x = 0$ to $x = 2$.
   (b) $f(x) = \ln(\cos(x))$ from $x = 0$ to $x = \pi/3$.
   (c) $f(x) = e^x$ from $x = 0$ to $x = 1$.

4. Set up a function $s(t)$ that gives the arc length of the curve $f(x) = 2x + 1$ from $x = 0$ to $x = t$. Find $s(4)$.

5. Compute the surface areas of revolution about the x-axis over the given interval for the following functions.
   (a) $y = x$, $[0, 4]$
   (b) $y = x^3$, $[0, 2]$
   (c) $y = \left(4 - x^{2/3}\right)^{3/2}$, $[0, 8]$
   (d) $y = e^{-x}$, $[0, 1]$
   (e) $y = \frac{1}{4}x^2 - \frac{1}{2}\ln x$, $[1, e]$
   (f) $y = \sin x$, $[0, \pi]$
   (g) Find the surface area of the torus obtained by rotating the circle $x^2 + (y - b)^2 = r^2$ about the x-axis.
   (h) Show that the surface area of a right circular cone of radius r and height h is $\pi r\sqrt{r^2 + h^2}$.
   Hint: Rotate a line $y = mx$ about the x-axis for $0 \le x \le h$, where m is determined by the radius r.

MA 114 Worksheet #21: Centers of Mass

1. Find the center of mass for the system of particles of masses 4, 2, 5, and 1 located at the coordinates $(1, 2)$, $(-3, 2)$, $(2, -1)$, and $(4, 0)$.
2. Point masses of equal size are placed at the vertices of the triangle with coordinates $(3, 0)$, $(b, 0)$, and $(0, 6)$, where $b > 3$. Find the center of mass.
3. Find the centroid of the region under the graph of $y = 1 - x^2$ for $0 \le x \le 1$.
4. Find the centroid of the region under the graph of $f(x) = \sqrt{x}$ for $1 \le x \le 4$.
5. Find the centroid of the region between $f(x) = x - 1$ and $g(x) = 2 - x$ for $1 \le x \le 2$.

MA 114 Worksheet #22: Parametric Curves

1. (a) How is a curve different from a parametrization of the curve?

   (b) Suppose a curve is parameterized by $(x(t), y(t))$ and that there is a time t_0 with $x'(t_0) = 0$, $x''(t_0) > 0$, and $y'(t_0) > 0$. What can you say about the curve near $(x(t_0), y(t_0))$?

   (c) What parametric equations represent the circle of radius 5 with center $(2, 4)$?

   (d) Represent the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{c^2} = 1$ with parametric equations.

   (e) Do the two sets of parametric equations

   $$y_1(t) = 5\sin(t),\ x_1(t) = 5\cos(t),\ 0 \le t \le 2\pi$$

   and

   $$y_2(t) = 5\sin(t),\ x_2(t) = 5\cos(t),\ 0 \le t \le 20\pi$$

   represent the same parametric curve? Discuss.

2. Consider the curve parametrized by $c(t) = (\sin(t) + \frac{t}{\pi}, \left(\frac{t}{\pi}\right)^2)$, for $0 \le t \le 2\pi$.

   (a) Plot the points given by $t = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{3\pi}{2}, 2\pi$.

   (b) Consider the derivatives of $x(t)$ and $y(t)$ when $t = \frac{\pi}{2}$ and $t = \frac{3\pi}{2}$. What does this tell you about the curve near these points?

   (c) Use the above information to plot the curve.

3. Find a Cartesian equation for the following parametric curves. Sketch the curves to see if you solved them correctly.

   (a) $x = \sqrt{t}$, $y = 1 - t$.

   (b) $x = 3t - 5$, $y = 2t + 1$.

   (c) $x = \cos(t)$, $y = \sin(t)$.

4. Represent each of the following curves as parametric equations traced just once on the indicated interval.

   (a) $y = x^3$ from $x = 0$ to $x = 2$.

   (b) $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$.

5. A particle travels from the point $(2, 3)$ to $(-1, -1)$ along a straight line over the course of 5 seconds. Write down a set of parametric equations which describe the position of the particle for any time between 0 and 5 seconds.

6. For the following parametric curves, find an equation for the tangent to the curve at the specified value of the parameter.

   (a) $x = e^{\sqrt{t}}$, $y = t - \ln(t^2)$ at $t = 1$.

(b) $x = \cos(\theta) + \sin(2\theta)$, $y = \cos(\theta)$, at $\theta = \pi/2$.

7. For the following parametric curve, find dy/dx.
   (a) $x = e^{\sqrt{t}}$, $y = t + e^{-t}$.
   (b) $x = t^3 - 12t$, $y = t^2 - 1$.
   (c) $x = 4\cos(t)$, $y = \sin(2t)$.

8. Find d^2y/dx^2 for the curve $x = 7 + t^2 + e^t$, $y = \cos(t) + \frac{1}{t}$, $0 < t \leq \pi$.

9. Find the arc length of the following curves.
   (a) $x = 1 + 3t^2$, $y = 4 + 2t^3$, $0 \leq t \leq 1$.
   (b) $x = 4\cos(t)$, $y = 4\sin(t)$, $0 \leq t \leq 2\pi$.
   (c) $x = 3t^2$, $y = 4t^3$, $1 \leq t \leq 3$.

10. Suppose you wrap a string around a circle. If you unwind the string from the circle while holding it taut, the end of the string traces out a curve called the *involute* of the circle. Suppose you have a circle of radius r centered at the origin, with the end of the string all the way wrapped up resting at the point $(r, 0)$. As you unwrap the string, define θ to be the angle formed by the x-axis and the line segment from the center of the circle to the point up to which you have unwrapped the string.
    (a) Draw a picture and label θ.
    (b) Show that the parametric equations of the involute are given by $x = r(\cos\theta + \theta\sin\theta)$, $y = r(\sin\theta - \theta\cos\theta)$.
    (c) Find the length of the involute for $0 \leq \theta \leq 2\pi$.

MA 114 Worksheet #23: Polar coordinates

1. Convert from rectangular to polar coordinates:
   (a) $(1, \sqrt{3})$
   (b) $(-1, 0)$
   (c) $(2, -2)$
2. Convert from polar to rectangular coordinates:
   (a) $\left(2, \frac{\pi}{6}\right)$
   (b) $\left(-1, \frac{\pi}{2}\right)$
   (c) $\left(1, -\frac{\pi}{4}\right)$
3. List all the possible polar coordinates for the point whose polar coordinates are $(-2, \pi/2)$.
4. Sketch the graph of the polar curves:
   (a) $\theta = \frac{3\pi}{4}$
   (b) $r = \pi$
   (c) $r = \cos\theta$
   (d) $r = \cos(2\theta)$
   (e) $r = 1 + \cos\theta$
   (f) $r = 2 - 5\sin\theta$
5. Find the equation in polar coordinates of the line through the origin with slope $\frac{1}{3}$.
6. Find the polar equation for:
   (a) $x^2 + y^2 = 9$
   (b) $x = 4$
   (c) $y = 4$
   (d) $xy = 4$
7. Convert the equation of the circle $r = 2\sin\theta$ to rectangular coordinates and find the center and radius of the circle.
8. Find the distance between the polar points $(3, \pi/3)$ and $(6, 7\pi/6)$.

MA 114 Worksheet #24: Review for Exam 03

1. Find the volume of the following solids.
   (a) The solid obtained by rotating the region bounded by $y = x^2$ and $x = y^2$ about the x-axis,
   (b) The solid obtained by rotating the region bounded by $x = y^2$ and $x = 1$ about the line $x = 1$,
   (c) The solid obtained by rotating the region bounded by $y = 4x - x^2$ and $y = 3$ about the line $x = 1$,
   (d) The solid with circular base of radius 1 and cross-sections perpendicular to the base that are equilateral triangles.

2. Find the area of the surface of revolution obtained by rotating the given curve about the given axis.

   (a) $y = \sqrt{x+1}$, $0 \leq x \leq 3$; about x-axis, (b) $x = 3t^2$, $y = 2t^3$, $0 \leq t \leq 5$; about y-axis.

3. Compute the arc length of the following curves.

   (a) $x = a\cos^3\theta$, $y = a\sin^3\theta$, $0 \leq \theta \leq 2\pi$, (b) $y = \sqrt{2 - x^2}$, $0 \leq x \leq 1$.

4. Find the centroid of the region bounded by $y = \sqrt{x}$ and $y = x$.

5. Find the average value of the function bounded by $y = 3\sin(x) + \cos(2x)$ on the interval $[0, \pi]$.

6. Compute the slope of the tangent line to the curve in Problem 3(a) above, with $a = 8$, at the point $(1, \sqrt{3})$. Use this to determine an equation for the tangent line.

7. Consider the curve given by the parametric equations $(x(t), y(t)) = (t^2, 2t + 1)$.
   (a) Find the tangent line to the curve at $(4, -3)$. Put your answer in the form $y = mx + b$.
   (b) Find second derivative $\dfrac{d^2y}{dx^2}$ at $(x, y) = (4, -3)$. Is the curve concave up or concave down near this point?

MA 114 Worksheet #25: Calculus with polar coordinates

1. Find dy/dx for the following polar curves.

   (a) $r = 2\cos\theta + 1$ (b) $r = 1/\theta$ (c) $r = 2e^{-\theta}$

2. In each of the following, compute the slope of the tangent line at the given point. Then sketch the curve and the tangent line.

   (a) $r = \sin\theta$ at $\theta = \pi/3$. (b) $r = 1/\theta$ at $\theta = \pi/2$.

3. (a) Give the formula for the area of region bounded by the polar curve $r = f(\theta)$ from $\theta = a$ to $\theta = b$. Give a geometric explanation of this formula.

   (b) Give the formula for the length of the polar curve $r = f(\theta)$ from $\theta = a$ to $\theta = b$.

   (c) Use these formulas to establish the formulas for the area and circumference of a circle.

4. Find the slope of the tangent line to the polar curve $r = \theta^2$ at $\theta = \pi$.

5. Find the point(s) where the tangent line to the polar curve $r = 2 + \sin\theta$ is horizontal.

6. Find the area enclosed by one leaf of the curve $r = \sin 2\theta$.

7. Find the arc length of one leaf of the curve $r = \sin 2\theta$.

8. Find the area of the region bounded by $r = \cos\theta$ for $\theta = 0$ to $\theta = \pi/4$.

9. Find the area of the region that lies inside both the curves $r = \sqrt{3}\sin\theta$ and $r = \cos\theta$.

10. Find the area in the first quadrant that lies inside the curve $r = 2\cos\theta$ and outside the curve $r = 1$.

11. Find the length of the curve $r = \theta^2$ for $0 \le \theta \le 2\pi$.

12. Write down an integral expression for the length of the curve $r = \sin\theta + \theta$ for $0 \le \theta \le \pi$ but do not compute the integral.

13. Consider the sequence of circles, C_n, defined by the equations $x^2 + \left(y + \frac{1}{\sqrt{n}}\right)^2 = \frac{1}{n}$. Define a_n as the area of circle C_n and b_n as the area between circles C_n and C_{n+1}.

    (a) Sketch the picture of this infinite sequence of circles.

    (b) Does $\sum_{n=1}^{\infty} a_n$ converge?

    (c) Does $\sum_{n=1}^{\infty} b_n$ converge?

    (d) Define the circles D_n by the equations $x^2 + \left(y + \frac{1}{n}\right)^2 = \frac{1}{n^2}$ with d_n as the area of D_n. Does $\sum_{n=1}^{\infty} d_n$ converge?

MA 114 Worksheet #26: Conic Sections

1. The point in a lunar orbit nearest the surface of the moon is called perilune and the point farthest from the surface is called apolune. The Apollo 11 spacecraft was placed in an elliptical lunar orbit with perilune altitude 110 km and apolune altitude 314 km (above the moon). Find an equation of this ellipse if the radius of the moon is 1728 km and the center of the moon is at one focus.
2. Find an equation for the ellipse with foci $(1, 1)$ and $(-1, -1)$ and major axis of length 4.
3. Use parametric equations and Simpson's Rule with $n = 12$ to estimate the circumference of the ellipse $9x^2 + 4y^2 = 36$.
4. Find the area of the region enclosed by the hyperbola $4x^2 - 25y^2 = 100$ and the vertical line through a focus.
5. If an ellipse is rotated about its major axis, find the volume of the resulting solid.
6. Find the centroid of the region enclosed by the x-axis and the top half of the ellipse $9x^2 + 4y^2 = 36$.
7. Calculate the surface area of the ellipsoid that is generated by rotating an ellipse about its major axis.

MA 114 Worksheet #27: Differential equations

1. (a) Is $y = \sin(3x) + 2e^{4x}$ a solution to the differential equation $y'' + 9y = 50e^{4x}$? Explain why or why not.

   (b) Explain why every solution of $dy/dx = y^2 + 6$ must be an increasing function.

   (c) What does is mean to say that a differential equation is linear or nonlinear?

2. Find all values of α so that $y(x) = e^{\alpha x}$ is a solution of the differential equation $y'' + y' - 12y = 0$.

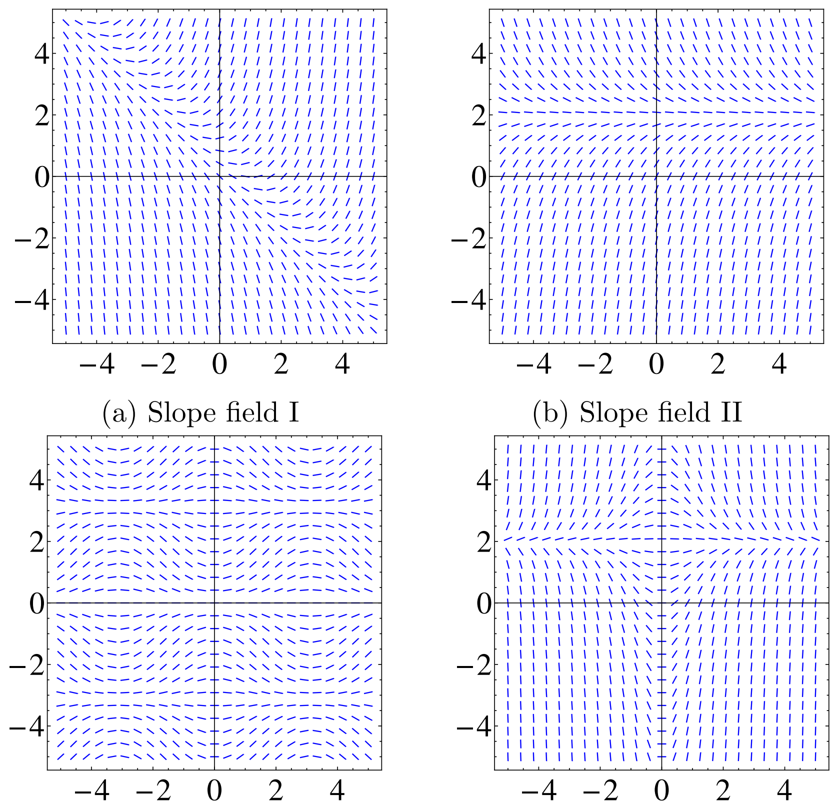
3. A tank has pure water flowing into it at 10 liters/min. The contents of the tank are kept thoroughly mixed, and the contents flow out at 10 liters/min. Salt is added to the tank at the rate of 0.1 kg/min. Initially, the tank contains 10 kg of salt in 100 liters of water. Formulate an initial value problem (that is, a differential equation along with initial conditions) whose solution is the quantity of salt in the tank at any time t. Do not solve the initial value problem.

4. Consider a tank with 200 liters of salt-water solution. A salt-water solution, with a concentration of 2 grams per liter, pours into the tank at a rate of 4 liters per minute. The well-mixed solution in the tank pours out at the same rate of 4 liters/minute. Write a differential equation expressing the rate of change in the concentration, $c(t)$, of salt in the tank. Do not solve.

MA 114 Worksheet #28: Direction fields, Separable Differential Equations

1. Match the differential equation with its slope field. Give reasons for your answer.

$$y' = 2 - y \qquad y' = x(2-y) \qquad y' = x + y - 1 \qquad y' = \sin(x)\sin(y)$$



(a) Slope field I

(b) Slope field II

(c) Slope Field III

(d) Slope field IV

Figure 1: Slope fields for Problem 1

2. Use slope field labeled IV to sketch the graphs of the solutions that satisfy the given initial conditions

$$y(0) = -1, \quad y(0) = 0, \quad y(0) = 1.$$

3. Sketch the slope field of the differential equation. Then use it to sketch a solution curve that passes through the given point

   (a) $y' = y^2$, $(1, 1)$

   (b) $y' = y - 2x$, $(1, 0)$

   (c) $y' = xy - x^2$, $(0, 1)$

4. Consider the autonomous differential equation $y' = y^2(3-y)(y+1)$. Without solving the differential equation, determine the value of $\lim_{t\to\infty} y(t)$, where the initial value is
   (a) $y(0) = 1$
   (b) $y(0) = 4$
   (c) $y(0) = -4$

5. Use Euler's method with step size 0.5 to compute the approximate y-values, y_1, y_2, y_3, and y_4 of the solution of the initial-value problem $y' = y - 2x$, $y(1) = 0$.

6. Use separation of variables to find the general solutions to the following differential equations.
   (a) $y' + 4xy^2 = 0$
   (b) $\sqrt{1-x^2}y' = xy$
   (c) $(1+x^2)y' = x^3y$
   (d) $y' = 3y - y^2$

MA 114 Worksheet #29: Review for Exam 04

This review worksheet covers only material discussed since Exam III.

To review for your final exam, be sure to study the material from Exams I, II, and III and the review sheets for these exams.

1. Identify and graph the conic section given by each of the following equations. Where applicable, find the foci.
   (a) $x^2 = 4y - 2y^2$
   (b) $x^2 + 3y^2 + 2x - 12y + 10 = 0$

2. Solve the initial value problem

$$\frac{dy}{dx} = \frac{x \sin x}{y}, \quad y(0) = -1$$

   It will help to know that

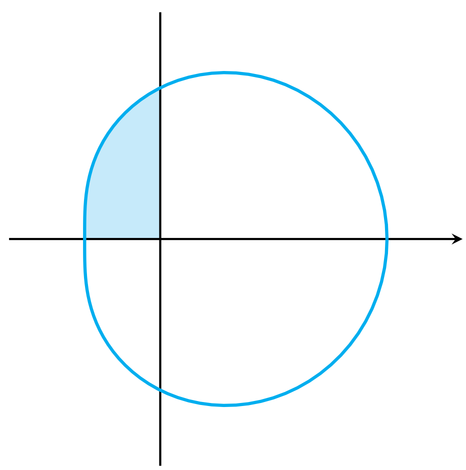
$$\int x \sin x \, dx = \sin x - x \cos x + C$$

3. By converting to Cartesian coordinates, identify and graph the curve $r^2 \sin 2\theta = 1$ (It may help to remember the identity $\sin 2\theta = 2 \sin\theta \cos\theta$).

4. Draw a direction field for the differential equation $y' = y(1-y)$. What are the equilibria? Classify each as stable or unstable.

5. Find the slope of the tangent line to the curve $r = 2\cos\theta$ at $\theta = \pi/3$.

6. Find the area of the region shown.



$r = 2 + \cos\theta$

7. Find the exact length of the polar curve $r = \theta^2$ for $0 \leq \theta \leq 2\pi$.
8. Use Euler's method with step size 0.1 to estimate $y(0.5)$, where $y(x)$ is the solution of the initial-value problem $y' = y + xy$, $y(0) = 1$.
9. Use Euler's method with step size 0.2 to estimate $y(1)$, where $y(x)$ is the solution of the initial-value problem $y' = x^2 y - \frac{1}{2}y^2$, $y(0) = 1$.
10. Solve the following differential equations.
    (a) $\dfrac{dy}{dx} = 3x^2y^2$
    (b) $xyy' = x^2 + 1$
    (c) $\dfrac{dy}{dx} + e^{x+y} = 0$
11. (a) Solve the differential equation $y' = 2x\sqrt{1 - y^2}$.
    (b) Solve the initial-value problem $y' = 2x\sqrt{1 - y^2}$, $y(0) = 0$, and graph the solution.
    (c) Does the initial-value problem $y' = 2x\sqrt{1 - y^2}$, $y(0) = 2$, have a solution? Explain.