Name: _____ Section: _____

Answer all questions and show your work. Unsupported answers may receive *no credit*. You may not use a calculator on this quiz. Allow 15 minutes for the quiz.

1. (5 points) Consider the polar curve $r = 2\sin(3\theta)$. Find the area enclosed by one leaf of the curve.

Solution: We have $r = 0$ at $\theta = 0$ and again when $3\theta = \pi$ or $\theta = \pi/3$. The area is given by $A = \int_0^{\pi/3} r(\theta)^2\, d\theta$. Evaluating this integral, we have

$$\begin{aligned}\int_0^{\pi/3} r(\theta)^2\, d\theta &= 4\int_0^{\pi/3} \sin^2(3\theta)\, d\theta \\ &= 4\int_0^{\pi/3} \frac{1-\cos(6\theta)}{2}\, d\theta \\ &= 2\left(\theta - \frac{\sin(6\theta)}{6}\right)\Bigg|_0^{\pi/3} \\ &= \frac{2\pi}{3}.\end{aligned}$$

(Limits of integration (1 point), integrand (2 points), use of double-angle formula (1 point), answer (1 point))

2. Consider the ellipse $\dfrac{(x-1)^2}{4} + \dfrac{(y+2)^2}{9} = 1$.
   (a) (3 points) Find the vertices of the ellipse.
   (b) (2 points) Give the lengths of the major and minor axes.

Solution: a) We obtain the given ellipse by translating the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ by $(1, -2)$. The vertices of the original ellipse are $(0, \pm 3)$. We add the point $(1, -2)$ to obtain the shifted vertices. They are

$$(1, -2 \pm 3) = (1, -5) \text{ and } (1, 1).$$

(2 points for one vertex, 1 point for second vertex)

b) The major axis is of length 6 and the minor axis is of length 4. (1 point each)