Name: ____________________ Section: ____________________

Answer all questions and show your work. Unsupported answers may receive *no credit*. You may not use a calculator on this quiz. Allow 15 minutes for the quiz.

1. (3 points) Find the anti-derivative. $\int \sin(x)\cos^2(x)\,dx$.

Solution: We substitute $u = \cos(x)$ and $du = -\sin(x)\,dx$ (1 point) to write

$$\begin{aligned}\int \sin(x)\cos^2(x)\,dx &= -\int u^2\,du && \text{(1 point)}\\ &= -\frac{u^3}{3} + C\\ &= -\frac{1}{3}\cos^3(x) + C && \text{(1 point)}\end{aligned}$$

Remark: We may check by differentiating.

2. (7 points) Find the anti-derivative. $\int \frac{1}{(9-x^2)^{3/2}}\,dx$

Solution: We use the trig substitution $x = 3\sin(u)$ and thus $dx = 3\cos(u)\,du$ (1 point) and $3\cos(u) = \sqrt{9-x^2}$ (1 point) to obtain

$$\begin{aligned}\int \frac{1}{(9-x^2)^{3/2}}\,dx &= \int \frac{3\cos(u)}{(3\cos(u))^3}\,du\\ &= \int \frac{1}{9\cos^2(u)}\,du + C && \text{(2 points)}\\ &= \frac{1}{9}\int \sec^2(u)\,du + C\\ &= \frac{1}{9}\tan(u) + C && \text{(1 point)}\\ &= \frac{x}{9\sqrt{9-x^2}} + C. && \text{(2 points)}\end{aligned}$$

The last equality holds since

$$\tan(u) = \frac{3\sin(u)}{3\cos(u)} = \frac{x}{\sqrt{9-x^2}}.$$