Name: ______________________ Section: ______________________

Answer all questions and show your work. Unsupported answers may receive *no credit*. You may not use a calculator on this quiz. Allow 15 minutes for the quiz.

1. (7 points) Evaluate the integral $\int_0^2 \frac{1}{x^2-2x-3}\,dx$.

Solution: We write $x^2 - 2x - 3 = (x-3)(x+1)$. Thus, we have a partial fractions decomposition

$$\frac{1}{x^2-2x-3} = \frac{A}{x+1} + \frac{B}{x-3} = \frac{(A+B)x + (B-3A)}{(x+1)(x-3)}.$$

This gives that $A + B = 0$ and $B - 3A = 1$. (1 point for form of decomposition, 2 points for coefficients) Solving these equations gives $B = 1/4$ and $A = -1/4$. Thus we have

$$\int \frac{1}{x^2-2x-3}\,dx = \frac{1}{4}\int\left(\frac{1}{x-3} - \frac{1}{x+1}\right)dx = \frac{1}{4}(\ln(|x-3|) - \ln(|x+1|)) + C.$$

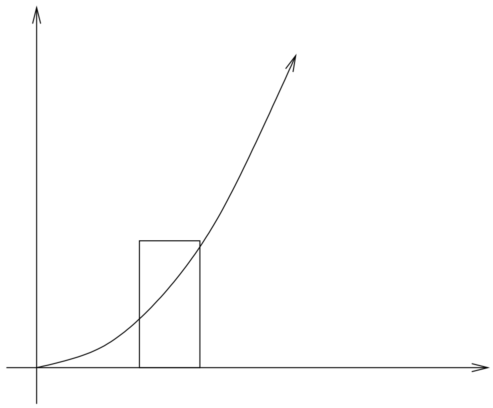
Using this anti-derivative, the definite integral becomes

$$\int_0^2 \frac{1}{x^2-2x-3}\,dx = \frac{1}{4}(\ln(1) - \ln(3) - (\ln(3) - \ln(1))) = -\frac{1}{2}\ln(3).$$

(2 points for anti-derivative, 1 point for evaluating and 1 point for simplifying.)

2. (3 points) Let R_n be a right sum for the integral $\int_0^4 x^6\,dx$. Is R_n larger or smaller than the exact value of the integral? Use a sketch to explain your answer.

Solution: Since the function x^6 is increasing on $[0, 4]$, the largest value of x^6 on a subinterval of this interval will be at the right endpoint. Thus, any right sum will be larger than the integral. The sketch below compares the area representing one term in a right sum with the corresponding integral.



Grading: (1 point) for correct answer. (2 points) for a sketch showing at least one term from a right sum.