Name: ______________________ Section: ______________________

Answer all questions and show your work. Unsupported answers may receive *no credit.* You may not use a calculator on this quiz. Allow 15 minutes for the quiz.

1. (a) (2 points) State the limit comparison test for convergence of series.

   (b) (3 points) Use the limit comparison test to compare with a p-series and determine if the series

$$\sum_{n=1}^{\infty} \frac{n^2}{3n^4+10}$$

converges or diverges.

Solution: a) Suppose $\sum a_n$ and $\sum b_n$ are series with positive terms. If $\lim_{n\to\infty} a_n/b_n = c$ with $0 < c < \infty$, then either both series converge or both series diverge. (2 points, deduct 1 point for minor errors) b) If we let $a_n = n^2/(3n^4+10)$ and $b_n = 1/n^2$, then $\lim_{n\to\infty} a_n/b_n = 1/3$. Since the series $\sum 1/n^2$ converges, then the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{3n^4+10}$ converges. (1 point compare with $1/n^2$), (1 point) show that the limit of a_n/b_n is nonzero, (1 point) give correct conclusion.)

2. (5 points) Use the integral test to find a value of N so that

$$\sum_{k=N+1}^{\infty} \frac{1}{k^3} \leq \frac{1}{50}.$$

Solution: We have $\displaystyle\sum_{k=N+1}^{\infty} \frac{1}{k^3} \leq \int_N^{\infty} \frac{1}{x^3}\,dx$. (compare with integral 1 point, correct endpoint 1 point)

Evaluating $\displaystyle\int_N^{\infty} \frac{1}{x^3}\,dx = \lim_{t\to\infty} \int_N^t \frac{1}{x^3}\,dx = \lim_{t\to\infty} \left.\frac{-1}{2x^2}\right|_N^t = 1/(2\cdot N^2)$ (value of improper integral 1 point). Solving $1/(2\cdot N^2) \leq 1/50$ gives $N \geq 5$ (solving inequality 1 point).