Name: ______________________ Section: ______________________

Answer all questions and show your work. Unsupported answers may receive *no credit*. You may not use a calculator on this quiz. Allow 15 minutes for the quiz.

1. For each of the following series compute the ratio $\lim_{n\to\infty} |a_{n+1}/a_n|$ and determine if the ratio test gives convergence, divergence, or no information.

   (a) (2 points) $\sum_{n=1}^{\infty} \frac{1}{n^2}$

   (b) (2 points) $\sum_{n=1}^{\infty} \frac{2^n}{n^{42}}$

Solution: a) The limit of the ratio a_{n+1}/a_n is $\lim_{n\to\infty} \frac{n^2}{(n+1)^2} = 1$ and the test gives no information. b) the limit of the ratio is $\lim_{n\to\infty} \frac{2^{n+1}}{(n+1)^{42}} \frac{n^{42}}{2^n} = 2$ and the ratio test implies the series diverges. (1 point for limit, 1 point for conclusion in each part)

2. (6 points) Give the interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{2^n x^n}{n^2}$.

Solution: Applying the ratio test, we find $\frac{2^{n+1}x^{n+1}}{(n+1)^2} \frac{n^2}{2^n x^n} = 2x \frac{n^2}{(n+1)^2}$ and $\lim_{n\to\infty} 2x \frac{n^2}{(n+1)^2} = 2x$ (2 points). Thus the series will converge if $|2x| < 1$ and diverges if $|2x| > 1$. The radius of convergence is $1/2$ (1 point). At the endpoints $x = \pm 1/2$, $(2x)^n/n^2 = (\pm 1)^n/n^2$. The series $\sum_{n=1}^{\infty} 1/n^2$ is a convergent p-series and since $|(-1)^n/n^2| = 1/n^2$, the series $(-1)^n/n^2$ is absolutely convergent. Thus the power series converges at both endpoints. The interval of convergence is $[-1/2, 1/2]$. (1 point for each endpoint, 1 point for final answer).

One may also use the alternating series test to obtain convergence at the left endpoint.