Name: ————————————————   Section: ————————————————

    Answer all questions and show your work. Unsupported answers may receive *no credit*. You may not use a calculator on this quiz. Allow 15 minutes for the quiz.

1. (4 points) Find the average value of the function $f(x) = \sin(x/2)$ on the interval $[0, \pi]$.

> **Solution:** The average value is $\dfrac{1}{\pi} \displaystyle\int_0^\pi \sin(x/2)\,dx = \dfrac{1}{\pi}(-2\cos(x/2))\Big|_0^\pi = \dfrac{2}{\pi}$.

2. (6 points) Consider the region in the plane $R = \{(x, y) : 0 \le y \le x^2,\ 0 \le x \le 2\}$.

   (a) (1 point) We rotate R about the x-axis to obtain a solid S. We slice S by the plane perpendicular to the x-axis which contains $x = a$. Let $A(a)$ be the area of the resulting cross-section.

       Find a formula for $A(a)$ when $0 \le a \le 2$.

   (b) (2 points) Express the volume of the solid S as an integral. (Do not evaluate the integral.)

   (c) (3 points) Let T be the solid obtained by rotating R about the line $y = -1$. Express the volume of T as an integral. (Do not evaluate the integral.)

> **Solution:** a) The cross section is a disk with radius a^2. Its area is πa^4.
>
> b) The volume is given by the integral of the function $A(x)$ found in part a). $\displaystyle\int_0^2 A(x)\,dx = \pi \int_0^2 x^4\,dx$. (2 points) If the answer to part b) includes an answer to part a), award the point for part a) here.
>
> c) The cross-section at $x = a$ is a washer with outer radius $1 + a^2$ and with a hole of radius 1. The area of this disk is $A(a) = \pi((1 + a^2)^2 - 1)$ (2 points). Integrating gives the volume as $\pi \displaystyle\int_0^2 ((1 + x^2)^2 - 1)\,dx$. (1 point)