Name: _____ Section: _____

Answer all questions and show your work. Unsupported answers may receive *no credit*. You may not use a calculator on this quiz. Allow 15 minutes for the quiz.

1. (4 points) Use the method of cylindrical shells to find the volume generated by rotating the region R bounded by $y = \sqrt{x}$, $x = 0$ and $y = 2$, about the y-axis.

Solution: The curves $y = 2$ and $y = \sqrt{x}$ intersect at $(4, 2)$. The volume of the solid is

$$\int_0^4 2\pi x(2 - \sqrt{x})\,dx = 2\pi\left(x^2 - \frac{2}{5}x^{5/2}\right)\Big|_0^4 = \frac{32}{5}\pi.$$

(radius of shell (1 point), height (1 point), limits of integration (1 point), answer (1 point))

2. (6 points) Consider the curve $y = x^3$, $0 \le x \le 1$.
   (a) (2 points) Express the arc length of the curve as an integral. (Do not evaluate the integral.)
   (b) (2 points) We rotate the curve about the x-axis to obtain a surface C. Express the surface area of C as an integral.
   (c) (2 points) Find the exact surface area of C.

Solution: (a) The arc length is $L = \displaystyle\int_0^1 \sqrt{1 + (3x^2)^2}\,dx$.

(b) The surface area is $S = \displaystyle\int_0^1 2\pi x^3\sqrt{1 + (3x^2)^2}\,dx$.

(c) Evaluating the integral from part (b) by the substitution $u = 1 + 9x^4$,

$$S = \int_0^1 2\pi x^3\sqrt{1 + 9x^4}\,dx = \frac{\pi}{18}\int_1^{10} u^{1/2}\,du = \frac{\pi}{18}\frac{2}{3}u^{3/2}\Big|_1^{10} = \frac{\pi}{27}\left(10^{3/2} - 1\right).$$