MA 114 Worksheet #28.5: Separable Differential Equations, review part 1

1. Use separation of variables to find the general solutions to the following differential equations.
   (a) $y' + 4xy^2 = 0$
   (b) $\sqrt{1-x^2}y' = xy$
   (c) $(1+x^2)y' = x^3y$
   (d) $y' = 3y - y^2$

2. Find the solution of the initial-value problem

$$\begin{cases} y' = y^2 - 4y \\ y(0) = 1 \end{cases}$$

   Find $\lim_{x\to\infty} y(x)$. Check your answer by considering a direction field.

3. Evaluate the following integrals.
   (a) $\displaystyle\int x\sin(x)\,dx$
   (b) $\displaystyle\int_0^{\sqrt{\pi}} x\sin(x^2)\,dx$
   (c) $\displaystyle\int \cos^2(x)\,dx$
   (d) $\displaystyle\int \cos^3(x)\,dx$
   (e) $\displaystyle\int \sqrt{4-x^2}\,dx$
   (f) $\displaystyle\int \frac{1}{x^3+2x}\,dx$

4. Consider the region $R = \{(x,y) : 0 \le y \le \sin(x), 2\pi \le x \le 3\pi\}$.
   (a) Suppose that we revolve R about the line $x = 1$. Write an integral that gives the volume of the solid of revolution.
   (b) Suppose that we revolve R about the line $y = -1$. Write an integral that gives the volume of the solid of revolution.

5. Consider the curve C with parametric equations $x = \sin(t)$, $y = \cos(t)$, for $0 \le t \le \pi/2$.
   (a) Write an integral for the length of the curve C and evaluate the integral to find the length.
   (b) The curve C is rotated about the x-axis. Find the surface area of the resulting surface of revolution.

6. Determine if each of the following series converges conditionally, converges absolutely or diverges.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n!}$

7. (a) Find a power series centered at zero for the function

$$F(x) = \int_0^x \sin(2t^2)\, dt.$$

(b) We want to find a partial sum of the series for $F(1/2)$ whose value is within 10^{-3} of the sum of the series. How many terms are needed? Hint: Use the alternating series test.