

Answer all questions and show your work. Unsupported answers may receive *no credit*. You may not use a calculator on this quiz. Allow 15 minutes for the quiz.

Name: \_\_\_\_\_ Section: \_\_\_\_\_

1. (5 points) Is the integral  $\int_1^{\infty} \frac{\ln x}{x} dx$  convergent or divergent? (Justify your answer!) If it is convergent compute the integral.

**Solution:** Use  $u$  substitution with  $u = \ln x$  and  $du = \frac{1}{x} dx$

$$\int \frac{\ln x}{x} dx = \int u du = \frac{1}{2} u^2 + C = \frac{1}{2} (\ln x)^2 + C$$

so

$$\int_1^{\infty} \frac{\ln x}{x} dx = \lim_{t \rightarrow \infty} \left( \frac{1}{2} (\ln x)^2 - \frac{1}{2} (\ln 1)^2 \right) = \lim_{t \rightarrow \infty} \frac{1}{2} (\ln x)^2$$

and this integral diverges.

2. Let  $a_n = \frac{n-1}{n+1}$ .

- (a) (2 points) What are the first 3 terms of the sequence?

**Solution:**  $a_0 = \frac{-1}{1} = -1$ ,  $a_1 = \frac{0}{2} = 0$ ,  $a_2 = \frac{2-1}{2+1} = \frac{1}{3}$ ,  $a_3 = \frac{2}{4} = \frac{1}{2}$ .

- (b) (3 points) What is the limit of the sequence?

**Solution:**

$$\frac{n-1}{n+1} = \frac{n+1-1-1}{n+1} = 1 + \frac{-2}{n+1} \mapsto 1$$