MA 114 Quiz 6 October 10, 2019

Answer all questions and show your work. Unsupported answers may receive *no credit*. You may not use a calculator on this quiz. Allow 15 minutes for the quiz.

Name: _____ Section: _____

1. (a) (5 points) Does the series $\sum_{n=3}^{\infty} \frac{n+2}{(n+1)^3}$ converge or diverge? Justify your answer!

Solution: We use the limit comparison test.

$$\lim_{n\to\infty} \frac{\frac{n+2}{(n+1)^3}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n+2}{(n+1)^3}\frac{n^2}{1} = \lim_{n\to\infty} \frac{n^3+2n^2}{n^3+3n^2+3n+1}$$
$$= \lim_{n\to\infty} \frac{1+\frac{2}{n}}{1+\frac{3}{n}+\frac{3}{n^2}+\frac{1}{n^3}} = 1$$

Since $\sum_{n=3}^{\infty} \frac{1}{n^2}$ (the exponent on n is > 1) converges the original series does too.

(b) (5 points) Does the series $\sum_{n=1}^{\infty} \frac{n!}{100^n}$ converge or diverge? Justify your answer!

Solution: We use the ratio test.

$$\lim_{n\to\infty} \frac{\frac{n!}{100^n}}{\frac{(n+1)!}{100^{n+1}}} = \lim_{n\to\infty} \frac{n!}{100^n}\frac{100^{n+1}}{(n+1)!} = \lim_{n\to\infty} \frac{100}{n+1}$$
$$= 0$$

Since this limit is less than 1 the series converges.