Answer all questions and show your work. Unsupported answers may receive *no credit*. You may not use a calculator on this quiz. Allow 15 minutes for the quiz.

Name: ____________________ Section: ____________________

1. (a) (5 points) What is the radius of convergence for $\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)2^n}(x-1)^n$? What is the interval of convergence?

**Solution:**

$$\left| \frac{\frac{(-1)^{n+1}}{(2(n+1)-1)2^{n+1}}(x-1)^{n+1}}{\frac{(-1)^n}{(2n-1)2^n}(x-1)^n} \right| = \left| \frac{(-1)^{n+1}(x-1)^{n+1}}{(2(n+1)-1)2^{n+1}} \frac{(2n-1)2^n}{(-1)^n(x-1)^n} \right|$$
$$= \left| \frac{(2n-1)}{2(2n+1)}(x-1) \right| \to \frac{1}{2}|x-1|$$

So the radius of convergence is 2.
Evaluating at $-1$,

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)2^n}((-1)-1)^n = \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)2^n}(-2)^n = \sum_{n=1}^{\infty} \frac{2^n}{(2n-1)2^n} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)}$$

This diverges by the integral test. Evaluating at 3,

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)2^n}(3-1)^n = \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)2^n}(2)^n = \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)}$$

and this converges by the alternating series test. So the interval of convergence is $(-1, 3]$.

(b) (5 points) Find a power series representation for $f(x) = \frac{x}{2x^2+1}$.

**Solution:** We can write $\frac{x}{2x^2+1}$ as $x\left(\frac{1}{1-(-2x^2)}\right)$. Since $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$, $\frac{1}{1-(-2x^2)} = \sum_{n=0}^{\infty}(-2x^2)^n = \sum_{n=0}^{\infty}(-2)^n x^{2n}$ and $f(x) = \sum_{n=0}^{\infty}(-2)^n x^{2n+1}$.