MA 114 Worksheet 24: Parametric Calculus

1. For the following parametric curve, find dy/dx.
   (a) $x = t^3 - 12t$, $y = t^2 - 1$.
   (b) $x = e^{\sqrt{t}}$, $y = t + e^{-t}$.
   (c) $x = 4\sin(t)$, $y = \cos(2t)$.

2. For the following parametric curves, find an equation for the tangent to the curve at the specified value of the parameter.
   (a) $x = e^{\sqrt{t}}$, $y = t - \ln(t^2)$ at $t = 1$.
   (b) $x = \cos(\theta) + \sin(2\theta)$, $y = \cos(\theta)$, at $\theta = \pi/2$.

3. Find d^2y/dx^2 for the curve $x = t^2 + e^t$, $y = \cos(t)$, $0 < t \leq \pi$.

4. Find the arc length of the following curves.
   (a) $x = 1 + 3t^2$, $y = 4 + 2t^3$, $0 \leq t \leq 1$.
   (b) $x = 4\cos(t)$, $y = 4\sin(t)$, $0 \leq t \leq 2\pi$.
   (c) $x = 3t^2$, $y = 4t^3$, $1 \leq t \leq 3$.

5. The equation of a circle of radius 3 is given by $x^2 + y^2 = 9$. Solving this equation for either x or y will give only half the circle. To resolve this, we will parameterize the circle using the angle θ counterclockwise from the x-axis as a variable.
   (a) Draw a picture of the circle and label θ.
   (b) Give the value of the point (x, y) at θ, in terms of θ. Hint: Recall what equations we use for the x and y coordinates on the unit circle. Make sure to account for the radius.
   (c) Give the parametric equations for x and y.
   (d) Find the arc length for $0 \leq \theta \leq 2$.

MA 114 Worksheet 25: Polar Coordinates I

1. Convert from rectangular to polar coordinates:
   (a) $(1, \sqrt{3})$
   (b) $(-1, 0)$
   (c) $(2, -2)$

2. Convert from polar to rectangular coordinates:
   (a) $\left(2, \frac{\pi}{6}\right)$
   (b) $\left(-1, \frac{\pi}{2}\right)$
   (c) $\left(1, -\frac{\pi}{4}\right)$

3. List all the possible polar coordinates for the point whose polar coordinates are $(-2, \pi/2)$.

4. Sketch the graph of the polar curves:
   (a) $\theta = \frac{3\pi}{4}$
   (b) $r = \pi$
   (c) $r = \cos\theta$
   (d) $r = \cos(2\theta)$
   (e) $r = 1 + \cos\theta$

5. Find the equation in polar coordinates of the line through the origin with slope $\frac{1}{3}$.

6. Find the polar equation for:
   (a) $x^2 + y^2 = 9$
   (b) $x = 4$
   (c) $xy = 4$
   (d) $x^2 + y^2 + 2x = 0$

7. Convert the equation of the circle $r = 2\sin\theta$ to rectangular coordinates and find the center and radius of the circle.

8. Find the distance between the polar points $(3, \pi/3)$ and $(6, 7\pi/6)$.

MA 114 Worksheet 26: Polar Coordinates II

1. Find dy/dx for the following polar curves.

   (a) $r = 2\cos\theta + 1$ (b) $r = 1/\theta$ (c) $r = 2e^{-\theta}$

2. In each of the following, compute the slope of the tangent line at the given point. Then sketch the curve and the tangent line.

   (a) $r = \sin\theta$ at $\theta = \pi/3$. (b) $r = 1/\theta$ at $\theta = \pi/2$.

3. (a) Give the formula for the area of region bounded by the polar curve $r = f(\theta)$ from $\theta = a$ to $\theta = b$. Give a geometric explanation of this formula.
   (b) Give the formula for the length of the polar curve $r = f(\theta)$ from $\theta = a$ to $\theta = b$.
   (c) Use these formulas to establish the formulas for the area and circumference of a circle.

4. Find the slope of the tangent line to the polar curve $r = \theta^2$ at $\theta = \pi$.

5. Find the point(s) where the tangent line to the polar curve $r = 2 + \sin\theta$ is horizontal.

6. Find the area enclosed by one leaf of the curve $r = \sin 2\theta$.

7. Find the arc length of the curve $r = e^{\theta}$ for $0 \leq \theta \leq 2\pi$.

8. Find the area of the region bounded by $r = \cos\theta$ for $0 \leq \theta \leq \pi/4$.

9. Find the area of the region that lies inside both the curves $r = \sqrt{3}\sin\theta$ and $r = \cos\theta$.

10. Find the area in the first quadrant that lies inside the curve $r = 2\cos\theta$ and outside the curve $r = 1$.

11. Write down an integral expression for the length of the curve $r = \sin\theta + \theta$ for $0 \leq \theta \leq \pi$ but do not compute the integral.

MA 114 Worksheet 27: Conic Sections

1. Find the slope of the tangent line to:

   a The parabola $y^2 = 6x$ at the point $\left(\frac{1}{2}, \sqrt{3}\right)$.

   b The ellipse $\frac{y^2}{2} + x^2 = 1$ at the point $\left(\frac{\sqrt{2}}{2}, 1\right)$.

2. Find the center and length of the major diameter for the ellipse $4x^2 + 2x + y^2 = 1$

3. Find the focus of the parabola $y = x^2 + 14x$.

4. Find the equation of the ellipse with foci at $(0, \pm 4)$ and vertices at $(\pm 3, 0)$.

5. Determine the distance D between the vertices of $-9x^2 + 18x + 4y^2 + 24y - 9 = 0$.

6. Find the equation of the parabola with vertex $(0, 0)$ and focus $\left(0, \frac{1}{4}\right)$.

7. Find the vertices and foci of the conic section $\frac{(x-7)^2}{5} - \frac{(y-3)^2}{5} = 1$.

MA 114 Worksheet 28: Review for Exam 4

This review worksheet covers only material discussed since Exam III.

To review for your final exam, be sure to study the material from Exams I, II, and III and the review sheets for these exams.

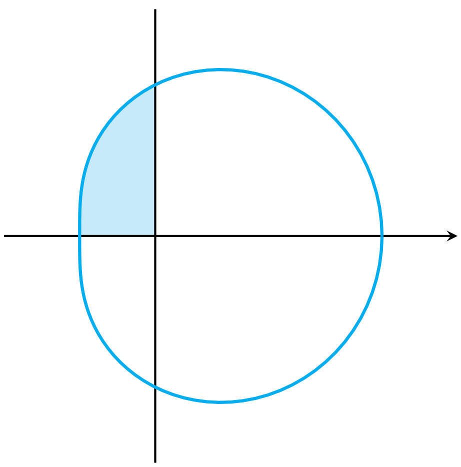
1. Consider the curve given by the parametric equations $(x(t), y(t)) = (t^2, 2t + 1)$.
   (a) Find the tangent line to the curve at $(4, -3)$. Put your answer in the form $y = mx + b$.
   (b) Find second derivative $\frac{d^2y}{dx^2}$ at $(x, y) = (4, -3)$. Is the curve concave up or concave down near this point?

2. Identify and graph the conic section given by each of the following equations. Where applicable, find the foci.
   (a) $x^2 = 4y - 2y^2$
   (b) $x^2 + 3y^2 + 2x - 12y + 10 = 0$

3. By converting to Cartesian coordinates, identify and graph the curve $r^2 \sin 2\theta = 1$ (It may help to remember the identity $\sin 2\theta = 2 \sin\theta \cos\theta$).

4. Find the slope of the tangent line to the curve $r = 2\cos\theta$ at $\theta = \pi/3$.

5. Find the area of the region shown.



$r = 2 + \cos\theta$

6. Find the exact length of the polar curve $r = \theta^2$ for $0 \le \theta \le 2\pi$.

7. Find the exact length of the polar curve $r = e^\theta$ for $0 \le \theta \le \pi$

8. Find a polar equation for the curve $(x - 1)^2 + y^2 = 2$.

9. Find the area enclosed by one loop of the lemniscate $r^2 = \cos(2\theta)$.

10. Compute the arc length of the following curves.

    (a) $x = a\cos^3\theta,\ y = a\sin^3\theta,\ 0 \le \theta \le 2\pi,$

    (b) $y = \sqrt{2 - x^2},\ 0 \le x \le 1.$