Name: _____ Section: _____

Answer all questions and show your work. Unsupported answers may receive *no credit*. You may not use a calculator on this quiz. Allow 15 minutes for the quiz.

1. (6 points) Find the exact length L of the polar curve

$$r = \theta^2, \quad 0 \le \theta \le \sqrt{21}.$$

Solution: We have

$$L = \int_0^{\sqrt{21}} \sqrt{r^2 + (dr/d\theta)^2}\, d\theta = \int_0^{\sqrt{21}} \sqrt{\theta^4 + 4\theta^2}\, d\theta = \int_0^{\sqrt{21}} \theta\sqrt{\theta^2 + 4}\, d\theta.$$

The substitution $u = \theta^2 + 4$ gives $du = 2\, d\theta$ and

$$\int \theta\sqrt{\theta^2 + 4}\, d\theta = \frac{1}{2}\int \sqrt{u}\, du = \frac{u^{3/2}}{3} + C = \frac{(\theta^2 + 4)^{3/2}}{3} + C.$$

Thus

$$L = \left[\frac{(\theta^2 + 4)^{3/2}}{3}\right]_{\theta=0}^{\theta=\sqrt{21}} = \frac{(25)^{3/2} - 4^{3/2}}{3} = \frac{125 - 8}{3} = 39.$$

2. (4 points) Find the vertex, focus, and directrix of the parabola $(x-1)^2 = 8y$.

Solution: First consider the parabola $x^2 = 8y$. Thus $4p = 8$ and $p = 2$. Therefore the vertex, focus, and directrix of $x^2 = 8y$ are, respectively,

$$(0,0), \quad (0,2), \quad \text{and} \quad y = -2.$$

The parabola $(x-1)^2 = 8y$ is obtained from the parabola $x^2 = 8y$ by translating it one unit to the right. Thus the vertex, focus, and directrix of $(x-1)^2 = 8y$ are, respectively,

$$(1,0), \quad (1,2), \quad \text{and} \quad y = -2.$$