Name: ______________________ Section: ______________________

Answer all questions and show your work. Unsupported answers may receive *no credit*. You may not use a calculator on this quiz. Allow 15 minutes for the quiz.

1. (4 points) Find the antiderivative $\int \sin^5(x)\,dx$.

Solution: Take $u = \cos(x)$. We have $du = -\sin(x)\,dx$ and

$$\sin^4(x) = (1-\cos^2(x))^2 = (1-u^2)^2 = 1-2u^2+u^4.$$

Thus

$$\begin{aligned}\int \sin^5(x)\,dx &= \int \sin^4(x)\sin(x)\,dx \\ &= -\int (1-2u^2+u^4)\,du \\ &= -u + \frac{2u^3}{3} - \frac{u^5}{5} + C \\ &= -\cos(x) + \frac{2\cos^3(x)}{3} - \frac{\cos^5(x)}{5} + C\end{aligned}$$

2. (6 points) Use a trigonometric substitution to evaluate the integral $\int_0^{1/2} \sqrt{1-4x^2}\,dx$.

Solution: We use the trigonometric substitution $x = \frac{1}{2}\sin(\theta)$ with $-\pi/2 \le \theta \le \pi/2$. Then $dx = \frac{1}{2}\cos(\theta)\,d\theta$ and $\sqrt{1-4x^2} = \cos(\theta)$. Thus

$$\begin{aligned}\int_0^{1/2} \sqrt{1-4x^2}\,dx &= \frac{1}{2}\int_0^{\pi/2} \cos^2(\theta)\,d\theta \\ &= \frac{1}{4}\int_0^{\pi/2} (1+\cos(2\theta))\,d\theta \\ &= \frac{1}{4}\left[\theta + \frac{\sin(2\theta)}{2}\right]_0^{\pi/2} \\ &= \frac{\pi}{8}.\end{aligned}$$