Name: ______________________ Section: ______________________

Answer all questions and show your work. Unsupported answers may receive *no credit.* You may not use a calculator on this quiz. Allow 15 minutes for the quiz.

1. (7 points) Use partial fractions to evaluate $\int \frac{x^3+1}{x^2-4}\,dx$.

Solution: First use the long division to write $\frac{x^3+1}{x^2-4}$ as a sum of a polynomial and a proper rational function:

$$\frac{x^3+1}{x^2-4} = x + \frac{4x+1}{x^2-4}.$$

Since $x^2-4=(x-2)(x+2)$ is a product of distinct linear terms, we write

$$\frac{4x+1}{x^2-4} = \frac{A}{x-2} + \frac{B}{x+2}.$$

Then

$$A(x+2) + B(x-2) = 4x+1,$$

i.e., $A+B=4$ and $2A-2B=1$. Solving this for A and B gives $A=9/4, B=7/4$. Thus

$$\frac{x^3+1}{x^2-4} = x + \frac{9}{4}\left(\frac{1}{x-2}\right) + \frac{7}{4}\left(\frac{1}{x+2}\right)$$

and

$$\int \frac{x^3+1}{x^2-4}\,dx = \frac{x^2}{2} + \frac{9}{4}\ln|x-2| + \frac{7}{4}\ln|x+2| + C.$$

2. (3 points) Let R_n be a right endpoint approximation for the integral $\int_1^5 e^{-x}\,dx$. Is R_n larger or smaller than the exact value of the integral?

Solution: Since the function e^{-x} is decreasing on $[1,5]$, the smallest value of e^{-x} on any subinterval of this interval will be at the right endpoint. Thus, any right endpoint sum will be smaller than the integral.