Name: ______________________ Section: ______________________

Answer all questions and show your work. Unsupported answers may receive *no credit.* You may not use a calculator on this quiz. Allow 15 minutes for the quiz.

1. (5 points) Use the Ratio Test to determine whether the series

$$\sum_{n=1}^{\infty} \frac{n^{10}\,10^n}{n!}$$

converges or diverges.

Solution: Letting $a_n = \dfrac{n^{10}\,10^n}{n!}$, we have

$$|a_{n+1}/a_n| = \left(\frac{(n+1)^{10}10^{n+1}}{(n+1)!}\right)\left(\frac{n!}{n^{10}\,10^n}\right) = \frac{10(n+1)^9}{n^{10}}.$$

Thus

$$\lim_{n\to\infty} |a_{n+1}/a_n| = \lim_{n\to\infty} \frac{10(n+1)^9}{n^{10}} = \lim_{n\to\infty} \left(\frac{10}{n}\right)\left(1+\frac{1}{n}\right)^9 = 0 < 1.$$

Thus the series converges absolutely by the Ratio Test.

2. (5 points) Determine the interval of convergence for the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}x^n}{3^n n}$.

Solution: Let $a_n = \dfrac{(-1)^{n-1}x^n}{3^n n}$. Then $|a_{n+1}/a_n| = \dfrac{n\,|x|}{3(n+1)}$. Thus

$$\lim_{n\to\infty} |a_{n+1}/a_n| = \lim_{n\to\infty} \frac{n\,|x|}{3(n+1)} = \frac{|x|}{3}.$$

Applying the Ratio Test, we see that the series $\displaystyle\sum_{n=1}^{\infty} a_n$ converges in $(-3,3)$. For $x = 3$, we have $a_n = \dfrac{(-1)^{n-1}}{n}$ and the series $\displaystyle\sum_{n=1}^{\infty} a_n$ converges by the Aternating Series Test. For $x = -3$, we have $a_n = \dfrac{1}{n}$ and the series $\displaystyle\sum_{n=1}^{\infty} a_n$ diverges since it is the p-series with $p = 1$ (harmonic series). Therefore the interval of convergence of $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}x^n}{3^n n}$ is $(-3,3]$.