

Name: \_\_\_\_\_ Section: \_\_\_\_\_

Answer all questions and show your work. Unsupported answers may receive *no credit*. You may not use a calculator on this quiz. Allow 15 minutes for the quiz.

1. (5 points) Find the average value of the function  $f(x) = xe^x$  on the interval  $[0, 2]$ .

**Solution:** Since

$$\int xe^x dx = xe^x - \int e^x dx = xe^x - e^x + C,$$

the average value of  $f(x)$  is

$$\frac{1}{2} \int_0^2 xe^x dx = \left[ \frac{xe^x - e^x}{2} \right]_{x=0}^{x=2} = \frac{e^2 + 1}{2}.$$

2. (5 points) Calculate the volume of the following solid  $S$ . The base is the region enclosed by  $y = x^4$  and  $y = 9$ . The cross-sections perpendicular to the  $y$ -axis are squares.

**Solution:** Consider the plane perpendicular to the  $y$ -axis which contains  $y = a$ . The area  $A(a)$  of the corresponding square cross-section of  $S$  is  $A(a) = (2\sqrt[4]{a})^2 = 4\sqrt{a}$ . Thus the volume of  $S$  is

$$\int_0^9 A(y) dy = \int_0^9 4\sqrt{y} dy = \left[ \frac{8y^{3/2}}{3} \right]_{y=0}^{y=9} = 72.$$