Name: ______________________ Section: ______________________

Answer all questions and show your work. Unsupported answers may receive *no credit.* You may not use a calculator on this quiz. Allow 15 minutes for the quiz.

1. (6 points) Use the method of cylindrical shells to compute the volume V of the solid obtained by rotating the region bounded by $y = \ln(x)$, $y = 0$ and $x = e$, about the y-axis. Note: You must use cylindrical shells to get credit for this problem.

Solution: The curves $y = \ln(x)$ and $y = 0$ intersect at the point $(1, 0)$. Thus the volume is given by

$$V = \int_1^e (2\pi x)\ln(x)\,dx.$$

To evaluate this integral, we use integration by parts: Let $du = 2\pi x\,dx$, $v = \ln(x)$. Then $u = \pi x^2$, $dv = dx/x$ and

$$\int (2\pi x)\ln(x)\,dx = \pi x^2\ln(x) - \int \pi x\,dx = \pi x^2\ln(x) - \frac{\pi x^2}{2} + C.$$

Therefore

$$V = \pi\left[x^2\ln(x) - \frac{x^2}{2}\right]_{x=1}^{x=e} = \frac{\pi(e^2+1)}{2}.$$

2. (4 points) Consider the curve C: $y = 2\sqrt{x}$ $(0 \le x \le 3)$. Set up an integral for the area A of the surface obtained by rotating C about the x-axis. Do not evaluate the integral.

Solution: If $f(x) = 2\sqrt{x}$, we have $f'(x) = \dfrac{1}{\sqrt{x}}$. Thus

$$A = \int_0^3 2\pi f(x)\sqrt{1 + (f'(x))^2}\,dx = 4\pi\int_0^3 \sqrt{x}\sqrt{1 + \frac{1}{x}}\,dx = 4\pi\int_0^3 \sqrt{x+1}\,dx.$$