Name: ________________________ Section: ________________________

Answer all questions and show your work. Unsupported answers may receive *no credit*. You may not use a calculator on this quiz. Allow 15 minutes for the quiz.

1. (3 points) Give the partial fraction decomposition for the function $f(x) = \dfrac{1}{x^2+2x}$.

Solution: We factor $x^2+2x = x(x+2)$ and thus we know that there are constants A and B so that

$$\frac{1}{x^2+x} = \frac{A}{x} + \frac{B}{x+2} = \frac{(A+B)x+2A}{x^2+2x}.$$

Solving $A+B=0$ and $2A=1$ gives $A=1/2$ and $B=-1/2$ so the partial fractions decomposition is

$$\frac{1}{2}(\frac{1}{x} - \frac{1}{x+2}).$$

Form of decomposition with constants (1 point), equations for A and B (1 point), values of A and B (1 point). Other methods are possible and should receive equivalent credit.

2. (4 points) Give the form of the partial fraction decomposition for the function $g(x) = \dfrac{x^2}{(x^2-2x+1)(x^4-1)}$. **Do not solve for the coefficients.**

Solution: We factor $(x^2-2x+1)(x^4-1) = (x-1)^2(x^2-1)(x^2+1) = (x-1)^3(x+1)(x^2+1)$. The partial fraction decomposition has the terms

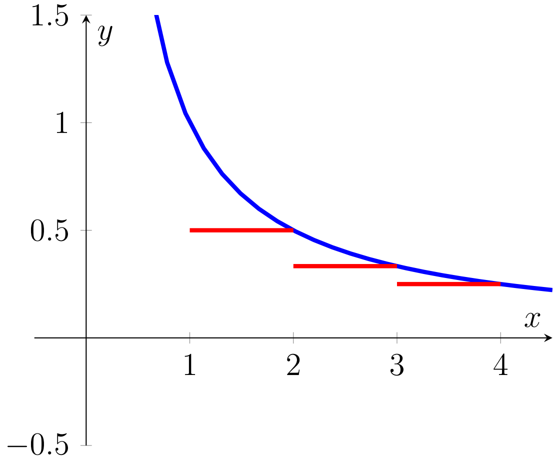
$$\frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} + \frac{D}{x+1} + \frac{Ex+F}{x^2+1}.$$

Factor demoninator (1 point), three terms involving $(x-1)$ (1 point), term involving $(x+1)$ (1 point), term involving (x^2+1) (1 point).

3. (3 points) (a) Find R_3, the right endpoint approximation to the integral $I = \displaystyle\int_1^4 \frac{1}{t}\,dt$.

   (b) Is the value R_3 greater or less than I? You may use a sketch to justify your answer.

Solution: a) We use three intervals of length 1 to give $R_3 = 1\cdot(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}) = \frac{13}{12}$. b) The rectangles whose area gives R_3 is contained in the area under the graph of $1/x$. Hence $R_3 < I$.



Value of R_3 (1 point), $R_3 < I$ (1 point), justification (1 point).