Name: ______________________ Section: ______________________

Answer all questions and show your work. Unsupported answers may receive *no credit*. You may not use a calculator on this quiz. Allow 15 minutes for the quiz.

1. Consider the sequence $\{a_n\}$ defined by $a_n = 3 \cdot 2^{-n}$ for $n = 1, 2, 3, \ldots$.
   (a) (2 points) Find the limit of the sequence $\lim_{n\to\infty} a_n$.
   (b) (4 points) Explain why the series $\sum_{n=1}^{\infty} a_n$ is convergent and find the sum.

Solution: a) We have $\lim_{n\to\infty} a_n = 0$.

b) This is a geometric series with ratio $r = 1/2$ and first term $a_1 = 3/2$. Since the ratio satisfies $|r| < 1$, the series is convergent and the sum is

$$\sum_{n=1}^{\infty} 3 \cdot 2^{-n} = \frac{a_1}{1-r} = \frac{3}{2} \cdot \frac{1}{1-1/2} = 3.$$

Grading: a) answer (2 points) b) ratio $r = 1/2$ (1 point), identify as convergent geometric series with ratio satisfying $|r| < 1$ (1 point), first term (1 point), answer (1 point).

Compare WeBWorK B2 #5.

2. Suppose that a recursive sequence is defined by $b_1 = 1$ and $b_n = \frac{1}{2}(b_{n-1} + \frac{3}{b_{n-1}})$.
   (a) (2 points) Find b_3.
   (b) (2 points) Suppose that the sequence b_n is convergent and $B = \lim_{n\to\infty} b_n$ exists. Find an equation that B satisfies.

Solution: a) We have $b_2 = (1+3/b_1)/2 = (1+3)/2 = 2$. Then $b_3 = (2+3/2)/2 = 7/4$ or 1.75.

b) Taking the limit of the relation $\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{2}(b_{n-1} + 3/b_{n-1})$ gives the equation

$$B = \frac{1}{2}(B + \frac{3}{B}).$$

Simplifying gives the equation $B^2 - 3 = 0$.

Grading: a) 1 point for b_2, 1 point for b_3. b) 2 points for a correct equation, simplification is not required.

Compare WeBWorK B1, #1, 5.