Name: ____________________ Section: ____________________

Answer all questions and show your work. Unsupported answers may receive *no credit*. You may not use a calculator on this quiz. Allow 15 minutes for the quiz.

1. For each of the series apply the ratio test and state if the series converges, diverges or if the ratio test gives no information.

   (a) (3 points) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n!}$.

   (b) (3 points) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n^{12}}$.

Solution: a) With $a_n = 2^n/n!$, we have $\dfrac{a_{n+1}}{a_n} = \dfrac{2^{n+1} \cdot n!}{2^n \cdot (n+1)!} = \dfrac{2}{n+1}$. The limit is $r = \lim\limits_{n\to\infty} \dfrac{2}{n+1} = 0$. Since $|r| < 1$, the series converges.

b) With $b_n = 2^n/n^{12}$, we have $\frac{b_{n+1}}{b_n} = \frac{2^{n+1} \cdot n^{12}}{2^n (n+1)^{12}} = 2\frac{n^{12}}{(n+1)^{12}}$. Evaluating the limit $r = \lim_{n\to\infty} 2\frac{n^{12}}{(n+1)^{12}} = 2$. Since $r > 2$, the series diverges.

Grading. On each part, give 1 point for simplifying ratio, 1 point for value of limit, 1 point for answer.

Based on WeBWorK B6 #1.

2. (4 points) Use the formula for the sum of a geometric series $\displaystyle\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$ to find a power series which equals $\dfrac{1}{1+x^2}$ for x in an interval containing 0.

   Write the terms involving x^n for $0 \le n \le 4$ without using summation notation.

Solution: We may write $\frac{1}{1+x^2} = \frac{1}{1-(-x^2)}$ and conclude that this is the sum of a geometric series with ratio $-x^2$. Thus $\dfrac{1}{1+x^2} = \displaystyle\sum_{n=0}^{\infty}(-x^2)^n = \sum_{n=0}^{\infty}(-1)^n x^{2n}$.

The first three terms are $1 - x^2 + x^4$.

Grading: Series (2 points). Three terms (2 points), give 1 point if at least one term is correct.

Compare WeBWorK B7 #7,8.