MA 114 Worksheet 00: Review of Calculus I

1. Find the derivatives of the following functions. Do not simplify your answers.
   (a) $y = x^8 + e^x + \dfrac{1}{x^2} + \sqrt[3]{x} + \ln x + 100$
   (b) $y = e^{4x^2} + \ln(3x+1)$
   (c) $y = (\sin x)e^x$
   (d) $y = \dfrac{x^2}{\cos(x^2)}$
2. Evaluate the following. These integrals shouldn't need substitutions.
   (a) $\displaystyle\int (x^3 + e^x)\,dx$
   (b) $\displaystyle\int \sqrt[5]{x} + \frac{1}{x^7}\,dx$
   (c) $\displaystyle\int \frac{1}{x}\,dx$
   (d) $\displaystyle\int \frac{1}{1+x^2}\,dx$
3. Evaluate these integrals, which might need substitutions.
   (a) $\displaystyle\int 3x^2\sin(x^3)\,dx$
   (b) $\displaystyle\int \sin(x)\cos(x)\,dx$
   (c) $\displaystyle\int \frac{6x+9}{x^2+3x+20}\,dx$
   (d) $\displaystyle\int \frac{1}{x\sqrt[5]{\ln x}}\,dx$
   (e) $\displaystyle\int \sec^2(x)\,dx$
   (f) $\displaystyle\int \frac{1}{(3x+10)^7}\,dx$
   (g) $\displaystyle\int \sec^2(x)e^{\tan x}\,dx$
   (h) $\displaystyle\int \frac{e^{1/x}}{x^2}\,dx$

MA 114 Worksheet 01: Integration by parts

1. Which of the following integrals should be solved using substitution and which should be solved using integration by parts?

   (a) $\displaystyle\int x\cos(x^2)\,dx$,

   (b) $\displaystyle\int e^x\sin(x)\,dx$,

   (c) $\displaystyle\int \frac{\ln(\arctan(x))}{1+x^2}dx$,

   (d) $\displaystyle\int xe^{x^2}\,dx$

2. Solve the following integrals using integration by parts:

   (a) $\displaystyle\int x^2\sin(x)\,dx$,

   (b) $\displaystyle\int (2x+1)e^x\,dx$,

   (c) $\displaystyle\int x\sin(3-x)\,dx$,

   (d) $\displaystyle\int \arctan(x)\,dx$,

   (e) $\displaystyle\int \ln(x)\,dx$

   (f) $\displaystyle\int x^5\ln(x)\,dx$

   (g) $\displaystyle\int e^x\cos(x)\,dx$

   (h) $\displaystyle\int x\ln(1+x)\,dx$ Hint: Make a substitution first, then try integration by parts.

3. Let $f(x)$ be a twice differentiable function with $f(1)=2$, $f(4)=7$, $f'(1)=5$ and $f'(4)=3$. Evaluate $\displaystyle\int_1^4 xf''(x)\,dx$

4. If $f(0)=g(0)=0$ and f'' and g'' are continuous, show that

$$\int_0^a f(x)g''(x)\,dx = f(a)g'(a) - f'(a)g(a) + \int_0^a f''(x)g(x)\,dx.$$

MA 114 Worksheet 02: Trigonometric Integrals Part A

1. Compute the following integrals:

   (a) $\displaystyle\int \sin(x)\sec^2(x)\,dx$

   (b) $\displaystyle\int_0^{2\pi} \sin^2\left(\tfrac{1}{3}\theta\right)\,d\theta$

   (c) $\displaystyle\int_0^{\pi/2} \cos^2(x)\,dx$

   (d) $\displaystyle\int \sqrt{\cos(x)}\sin^3(x)\,dx$

2. Compute the following integrals, which will require substitution.

   (a) $\displaystyle\int 2\ln(\sin(x))\cot(x)\,dx$

   (b) $\displaystyle\int \frac{e^{\sqrt{2x+1}}}{\sqrt{2x+1}}\,dx$

   (c) $\displaystyle\int (3x^2+2x-4)^3(6x+2)\,dx$

   (d) $\displaystyle\int \frac{\sin(\ln(x))}{x}\,dx$

3. Evaluate $\displaystyle\int \sin x\cos x\,dx$ by four methods:

   (a) the substitution $u=\cos(x)$;

   (b) the substitution $u=\sin(x)$;

   (c) the identity $\sin 2x = 2\sin(x)\cos(x)$;

   (d) integration by parts

   Explain the different appearances of the answers.

4. Find the area of the region bounded by the curves $y=\sin^2(x)$ and $y=\sin^3(x)$ for $0\le x\le\pi$.

MA 114 Worksheet 02: Trigonometric Integrals Part B

1. Compute the following integrals:

   (a) $\displaystyle\int_0^{\pi/2} (2-\sin(\theta))^2 \, d\theta$

   (b) $\displaystyle\int 4\sin^2(x)\cos^2(x)\,dx$

   (c) $\displaystyle\int \sin^3(x)\,dx$

   (d) $\displaystyle\int \cos^5(x)\,dx.$

   (e) $\displaystyle\int \cos(x)\sin(2x)\,dx.$

   (f) $\displaystyle\int \cos(x)\cos(2x)\,dx$

2. Find the anti-derivative $\int \cot(x)\,dx$. Hint: Substitute $u = \sin(x)$.

3. Compute the following integrals, which will require integration by parts.

   (a) $\displaystyle\int e^x \sin(x)\,dx$

   (b) $\displaystyle\int x\sin(2x)\,dx$

   (c) $\displaystyle\int x^2 \ln(3x)\,dx$

   (d) $\displaystyle\int \ln(\sin(x))\cos(x)\,dx$ (Hint: Make a substitution first, then try integration by parts.)

4. Find the area of the region bounded by the curves $y = \cos^2(x)$ and $y = \cos^3(x)$ for $0 \le x \le \pi$.

MA 114 Worksheet 03: Trig Substitution

1. Use the trigonometric substitution $x = \sin(u)$ to find $\displaystyle\int \frac{1}{\sqrt{1-x^2}}dx$.

2. Compute the following integrals:

   (a) $\displaystyle\int_0^2 \frac{u^3}{\sqrt{16-u^2}}du$

   (b) $\displaystyle\int \frac{1}{x^2\sqrt{25-x^2}}dx$

   (c) $\displaystyle\int \frac{x^2}{\sqrt{9-x^2}}\,dx$

   (d) $\displaystyle\int \frac{x^3}{\sqrt{4+x^2}}\,dx$

   (e) $\displaystyle\int \frac{1}{(1+x)^2}\,dx$

   (f) $\displaystyle\int_0^3 \frac{x}{\sqrt{36-x^2}}dx$

3. Evaluate the following integrals. One may be easily evaluated by substitution $u = 1+x^2$ and for the other use an appropriate trigonometric substitution.

$$\int \frac{\sqrt{1+x^2}}{x}dx \quad , \quad \int \frac{x}{\sqrt{1+x^2}}dx$$

4. (a) Evaluate the integral $\displaystyle\int_0^r \sqrt{r^2-x^2}dx$ using trigonometric substitution.

   (b) Use your answer to part a) to verify the formula for the area of a circle of radius r.

5. Let $r > 0$. Consider the identity

$$\int_0^s \sqrt{r^2-x^2}\,dx = \frac{1}{2}r^2 \arcsin\left(\frac{s}{r}\right) + \frac{1}{2}s\sqrt{r^2-s^2}$$

   where $0 \leq s \leq r$.

   (a) Plot the curves $y = \sqrt{r^2-x^2}$, $x = s$, and $y = \dfrac{x}{s}\sqrt{r^2-s^2}$.

   (b) Using part (a), verify the identity geometrically.

   (c) Verify the identity using trigonometric substitution.

MA 114 Worksheet 04: Integration by Partial Fractions

1. Write out the general form for the partial fraction decomposition but **do not** determine the numerical value of the coefficients.

   (a) $\dfrac{1}{x^2+3x+2}$

   (b) $\dfrac{x+1}{x^2+4x+4}$

   (c) $\dfrac{x}{(x^2+1)(x+1)(x+2)}$

   (d) $\dfrac{2x+5}{(x^2+1)^3(2x+1)}$

2. Based on your work in the previous question, can you conjecture (guess) a relation between the degree of the denominator of the rational function and the number of coefficients in the partial fraction decomposition?

3. Find the partial fraction decomposition for the following rational functions.

   (a) $\dfrac{1}{x^2-4}$

   (b) $\dfrac{x^3}{x^2-4}$ Hint: This is not a proper rational function, so you will begin by dividing.

   (c) $\dfrac{x}{(x^2+1)(x+1)}$

4. Compute the following integrals.

   (a) $\displaystyle\int \frac{x-9}{(x+5)(x-2)}\,dx$

   (b) $\displaystyle\int \frac{1}{x^2+3x+2}\,dx$

   (c) $\displaystyle\int \frac{x^3-2x^2+1}{x^3-2x^2}\,dx$

   (d) $\displaystyle\int \frac{x^3+4}{x^2+4}\,dx$

   (e) $\displaystyle\int \frac{1}{x(x^2+1)}\,dx$

5. Compute

$$\int \frac{1}{\sqrt{x}-\sqrt[3]{x}}\,dx$$

   by first making the substitution $u=\sqrt[6]{x}$.

MA 114 Worksheet 05: Numerical Integration

1. (a) Write down the Midpoint rule and illustrate how it works with a sketch.

   (b) Write down the Trapezoid rule, illustrate how it works with a sketch, and write down the error bound associated with it.

   (c) How large should n be in the Midpoint rule so that you can approximate

   $$\int_0^1 \sin(x)\,dx$$

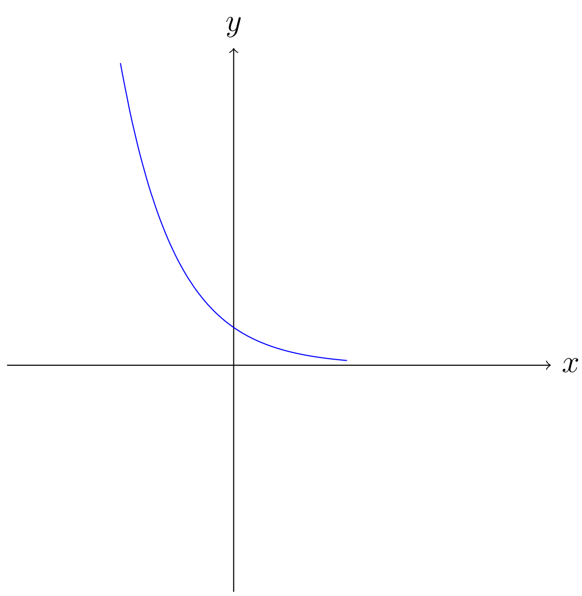
   with an error less than 10^{-7}?

2. Use the Midpoint rule to approximate the value of $\displaystyle\int_{-1}^{1} e^{-x^2}\,dx$ with $n = 4$. Draw a sketch to determine if the approximation is an overestimate or an underestimate of the integral.

3. The left, right, Trapezoidal, and Midpoint Rule approximations were used to estimate $\int_0^2 f(x)\,dx$, where f is the function whose graph is shown. The estimates were 0.7811, 0.8675, 0.8632, and 0.9540, and the same number of sub- intervals were used in each case.

   (a) Which rule produced which estimate?

   (b) Between which two approximations does the true value of $\int_0^2 f(x)\,dx$ lie?



4. Draw the graph of $f(x) = \sin\left(\frac{1}{2}x^2\right)$ in the viewing rectangle $[0, 1]$ by $[0, 0.5]$ and let $I = \int_0^1 f(x)\,dx$.

   (a) Use the graph to decide whether L_2, R_2, M_2, and T_2 underestimate or overestimate I.

(b) For any value of n, list the numbers L_n, R_n, M_n, T_n, and I in increasing order.

(c) Compute L_5, R_5, M_5, and T_5. From the graph, which do you think gives the best estimate of I?

5. The velocity in meters per second for a particle traveling along the axis is given in the table below. Use the Midpoint rule and Trapezoid rule to approximate the total distance the particle traveled from $t = 0$ to $t = 6$.

t	$v(t)$
0	0.75
1	1.34
2	1.5
3	1.9
4	2.5
5	3.2
6	3.0

MA 114 Worksheet 06: Simpson's Rule & Improper Integrals

1. (a) Write down Simpson's rule and illustrate how it works with a sketch.

   (b) How large should n be in Simpson's rule so that you can approximate
   $$\int_0^1 \sin x \, dx$$
   with an error less than 10^{-7}?

2. Approximate the integral
   $$\int_1^2 \frac{1}{x} \, dx$$
   using Simpson's rule. Choose n so that your error is certain to be less than 10^{-3}. Compute the exact value of the integral and compare to your approximation.

3. Simpson's Rule turns out to exactly integrate polynomials of degree three or less. Show that Simpson's rule gives the *exact* value of $\displaystyle\int_0^h p(x) \, dx$ where $h > 0$ and $p(x) = ax^3 + bx^2 + cx + d$. [Hint: First compute the exact value of the integral by direct integration. Then apply Simpson's rule with $n = 2$ and observe that the approximation and the exact value are the same.]

4. Explain why the following computation is wrong and determine the correct answer. (Try sketching or graphing the integrand to see where the problem lies.)
   $$\begin{aligned}\int_2^{10} \frac{1}{2x-8} dx &= \frac{1}{2} \int_{-4}^{12} \frac{1}{u} \, du \\ &= \frac{1}{2} \ln |x| \Big|_{-4}^{12} \\ &= \frac{1}{2} (\ln 12 - \ln 4)\end{aligned}$$
   where we used the substitution $u(x) = 2x - 8$.

5. A manufacturer of light bulbs wants to produce bulbs that last about 700 hours but, of course, some bulbs burn out faster than others. Let $F(t)$ be the fraction of the company's bulbs that burn out before t hours, so $F(t)$ always lies between 0 and 1.

   (a) Make a rough sketch of what you think the graph of F might look like.

   (b) What is the meaning of the derivative $r(t) = F'(t)$?

   (c) What is the value of $\int_0^\infty r(t) \, dt$? Why?

MA 114 Worksheet 07: Review for Exam 1

1. Find the following antiderivatives

    1. $\int x^2 \sin(2x)\,dx$
    2. $\int xe^{2x}\,dx$
    3. $\int \frac{dx}{x^2+2x+10}$
    4. $\int \frac{x+3}{(x-6)(x-3)}\,dx$
    5. $\int \frac{3x+6}{x^2-10x+24}\,dx$
    6. $\int \frac{3x^2+9x+8}{x^2(x+2)^2}\,dx$
    7. $\int \sin^5(x)\cos(x)\,dx$
    8. $\int \sin^2(x)\,dx$
    9. $\int \frac{dx}{x\sqrt{x^2+9}}$
    10. $\int \sqrt{16+4x^2}\,dx$
    11. $\int x^3\sqrt{9-x^2}\,dx$
    12. $\int_1^2 \frac{dx}{x\ln x}$
    13. $\int_1^\infty xe^{-2x}\,dx$

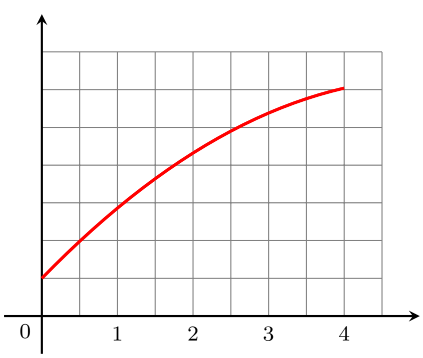
2. Let $f(x) = e^{-x^2}$. Find a value of N for use in the trapezoid rule to compute

$$\int_0^3 e^{-x^2}\,dx$$

   accurate to within 0.0001. Hint: $|f(x)| \le 1$ and $|f'(x)| \le 1$ on $[0,3]$.

3. Calculate M_6 and T_6 to approximate $\int_{-2}^{1} e^{x^2}\,dx$.

4. Let $I = \int_0^4 f(x)\,dx$, where f is the function whose graph is shown below. For any value of n, list the numbers L_n, R_n, M_n, and T_n in increasing order.



5. An airplane's velocity is recorded at 5-minute intervals during a 1 hour period with the following results, in miles per hour:

   550, 575, 600, 580, 610, 640, 625,
   595, 590, 620, 640, 640, 630

    1. Use Simpson's Rule to estimate the distance traveled during the hour.
    2. Use the trapezoid rule to estimate the distance traveled during the hour.