MA 114 Worksheet 25: Polar Coordinates I

1. Convert from rectangular to polar coordinates:
   (a) $(1, \sqrt{3})$
   (b) $(-1, 0)$
   (c) $(2, -2)$
2. Convert from polar to rectangular coordinates:
   (a) $\left(2, \frac{\pi}{6}\right)$
   (b) $\left(-1, \frac{\pi}{2}\right)$
   (c) $\left(1, -\frac{\pi}{4}\right)$
3. List all the possible polar coordinates for the point whose polar coordinates are $(-2, \pi/2)$.
4. Sketch the graph of the polar curves:
   (a) $\theta = \frac{3\pi}{4}$
   (b) $r = \pi$
   (c) $r = \cos\theta$
   (d) $r = \cos(2\theta)$
   (e) $r = 1 + \cos\theta$
   (f) $r = 2 - 5\sin\theta$
5. Find the equation in polar coordinates of the line through the origin with slope $\frac{1}{3}$.
6. Find the polar equation for:
   (a) $x^2 + y^2 = 9$
   (b) $x = 4$
   (c) $y = 4$
   (d) $xy = 4$
7. Convert the equation of the circle $r = 2\sin\theta$ to rectangular coordinates and find the center and radius of the circle.
8. Find the distance between the polar points $(3, \pi/3)$ and $(6, 7\pi/6)$.

MA 114 Worksheet 26: Polar Coordinates II

1. Find dy/dx for the following polar curves.

   (a) $r = 2\cos\theta + 1$ (b) $r = 1/\theta$ (c) $r = 2e^{-\theta}$

2. In each of the following, compute the slope of the tangent line at the given point. Then sketch the curve and the tangent line.

   (a) $r = \sin\theta$ at $\theta = \pi/3$. (b) $r = 1/\theta$ at $\theta = \pi/2$.

3. (a) Give the formula for the area of region bounded by the polar curve $r = f(\theta)$ from $\theta = a$ to $\theta = b$. Give a geometric explanation of this formula.
   (b) Give the formula for the length of the polar curve $r = f(\theta)$ from $\theta = a$ to $\theta = b$.
   (c) Use these formulas to establish the formulas for the area and circumference of a circle.

4. Find the slope of the tangent line to the polar curve $r = \theta^2$ at $\theta = \pi$.

5. Find the point(s) where the tangent line to the polar curve $r = 2 + \sin\theta$ is horizontal.

6. Find the area enclosed by one leaf of the curve $r = \sin(2\theta)$.

7. Find the arc length of the curve $r = 5^\theta$ for $\theta = 0$ to $\theta = 2\pi$.

8. Find the area of the region bounded by $r = \cos\theta$ for $\theta = 0$ to $\theta = \pi/4$.

9. Find the area of the region that lies inside both the curves $r = \sqrt{3}\sin\theta$ and $r = \cos\theta$.

10. Find the area in the first quadrant that lies inside the curve $r = 2\cos\theta$ and outside the curve $r = 1$.

11. Find the length of the curve $r = \theta^2$ for $0 \le \theta \le 2\pi$.

12. Write down an integral expression for the length of the curve $r = \sin\theta + \theta$ for $0 \le \theta \le \pi$ but do not compute the integral.

MA 114 Worksheet 27: Conic Sections

1. Find the slope of the tangent line to:

   a The parabola $y^2 = 6x$ at the point $\left(\frac{2}{3}, 2\right)$.

   b The ellipse $\frac{1}{2}y^2 + x^2 = 1$ at the point $\left(\frac{1}{3}\sqrt{6}, 1\right)$.

2. The ellipse $13x^2 + 2x + y^2 = 1$ has its center at the point (h, k) where

   $h =$

   $k =$

   The length of the major diameter of this ellipse is

3. The parabola $y = x^2 + 14x$ has its focus at the point (h, k) where

   $h =$

   $k =$

4. Find the equation of the ellipse with foci at $(0, \pm 10)$ and vertices at $(\pm 8, 0)$.

5. Determine the distance D between the vertices of $-9x^2 + 18x + 4y^2 + 24y - 9 = 0$.

6. Find the equation of the parabola with vertex $(0, 0)$ and focus $(8, 0)$.

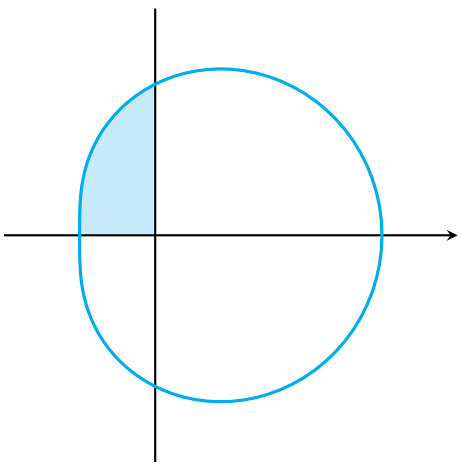
7. Find the vertices and foci of the conic section $\frac{(x-7)^2}{5} - \frac{(y-3)^2}{5} = 1$.

MA 114 Worksheet 28: Review for Exam 4

This review worksheet covers only material discussed since Exam III.

To review for your final exam, be sure to study the material from Exams I, II, and III and the review sheets for these exams.

1. Identify and graph the conic section given by each of the following equations. Where applicable, find the foci.
   (a) $x^2 = 4y - 2y^2$
   (b) $x^2 + 3y^2 + 2x - 12y + 10 = 0$
2. By converting to Cartesian coordinates, identify and graph the curve $r^2 \sin(2\theta) = 1$ (It may help to remember the identity $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$).
3. Find the slope of the tangent line to the curve $r = 2\cos(\theta)$ at $\theta = \pi/3$.
4. Find the area of the region shown.



$r = 2 + \cos\theta$

5. Find the exact length of the polar curve $r = \theta^2$ for $0 \leq \theta \leq 2\pi$.
6. Find the exact length of the polar curve $r = e^\theta$ for $0 \leq \theta \leq \pi$
7. Find a polar equation for the curve $(x-1)^2 + y^2 = 2$.
8. Find the area enclosed by one loop of the lemniscate $r^2 = cos(2\theta)$.