MA 114 — Calculus II Spring 2014
Exam 2 March 11, 2014

Name: ______________________________

Section: ______________________________

Last 4 digits of student ID #: ________

- No books or notes may be used.
- Turn off all your electronic devices and do not wear ear-plugs during the exam.
- You may use a calculator, but not one which has symbolic manipulation capabilities or a QWERTY keyboard.
- Additional blank sheets for scratch work are available upon request.
- **Multiple Choice Questions:** Record your answers on the right of this cover page by marking the box corresponding to the correct answer.
- **Free Response Questions:** Show all your work on the page of the problem. Show all your work. Clearly indicate your answer and the reasoning used to arrive at that answer.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E

Exam Scores

Question	Score	Total
MC		20
5		15
6		18
7		18
8		14
9		15
Total		100

Unsupported answers for the free response questions may not receive credit!

Record the correct answer to the following problems on the front page of this exam.

1. Which of the following is the correct form for the partial fraction decomposition of $\dfrac{4x^2+5}{(x-3)^2(2x+3)}$?

A. $\dfrac{Ax+B}{(x-3)^2} + \dfrac{C}{2x+3}$

B. $\dfrac{A}{(x-3)} + \dfrac{B}{(x-3)^2} + \dfrac{Cx+D}{(2x+3)}$

C. $\dfrac{A}{(x-3)} + \dfrac{B}{(x-3)^2} + \dfrac{C}{(2x+3)}$

D. $\dfrac{Ax+B}{(x-3)} + \dfrac{Cx+D}{(2x+3)}$

E. None of the above

2. Which of the following integrals represents the area of the surface obtained by revolving the curve $y = \cos(x)$ between $x = 0$ and $x = \pi/2$ about the x-axis?

A. $\int_0^{\pi/2} \pi \cos^2 x \, dx.$

B. $\int_0^{\pi/2} \sqrt{1+\sin^2 x} \, dx.$

C. $\int_0^{\pi/2} 2\pi \cos(x)\sqrt{1+\sin^2 x} \, dx.$

D. $\int_0^{\pi/2} \sqrt{1+\cos^2 x} \, dx.$

E. $\int_0^{\pi/4} 2\pi \sin(x)\sqrt{1+\cos^2 x} \, dx.$

Record the correct answer to the following problems on the front page of this exam.

3. Suppose that $y(t)$ satisfying the initial value problem

$$y' = 3(y-2)$$
$$y(0) = 4$$

Then:

A. $\lim_{t\to+\infty} y(t) = +\infty$

B. $\lim_{t\to+\infty} y(t) = 0$

C. $\lim_{t\to+\infty} y(t) = 4$

D. $\lim_{t\to+\infty} y(t) = 3$

E. $\lim_{t\to+\infty} y(t) = 5$

4. Which of the following is the Taylor polynomial of order 4 (expanding about $x = 0$) for the function $f(x) = \frac{1}{2}\left(e^x + e^{-x}\right)$?

A. $1 + x + \dfrac{x^2}{2} + \dfrac{x^3}{6} + \dfrac{x^4}{24}$

B. $1 - x + \dfrac{x^2}{2} - \dfrac{x^3}{6} + \dfrac{x^4}{24}$

C. $1 + x + \dfrac{x^3}{6}$

D. $1 + \dfrac{x^2}{2} + \dfrac{x^4}{24}$

E. $1 - \dfrac{x^2}{2} - \dfrac{x^4}{24}$

Free Response Questions: Show your work!

5. (15 points)

(a) (8 points) Find the partial fraction decomposition of

$$\frac{10}{(x-1)(x^2+9)}$$

Solution:
The final answer (2 points) is

$$\frac{10}{x-1} + \frac{-x-1}{x^2+9}$$

(b) (7 points) Evaluate the integral

$$\int \frac{2x-5}{x^2+1}\,dx$$

Solution:
$\ln(x^2+1) - 5\arctan(x) + C$

Free Response Questions: Show your work!

6. (18 points) Determine whether each of the following improper integrals is convergent or divergent. If the integral converges, evaluate the integral.

(a) (9 points) $\int_0^\infty \frac{1}{1+x}\, dx$

Solution:
Divergent since $\int_0^R \frac{1}{1+x}\, dx = \ln(1+R)$

(b) (9 points) $\int_1^2 \frac{1}{x(\ln x)^2}\, dx$

Solution:
Divergent since $\int_a^2 \frac{1}{x \ln x}\, dx = \ln(\ln(2) - \ln(\ln(a))$

Free Response Questions: Show your work!

7. (18 points) Find the area of the surface of revolution obtained by revolving

$$y = x^3$$

on the interval $[0, 2]$ about the x-axis.

Solution:
The integral is

$$\int_0^2 2\pi x^3 \cdot \sqrt{1 + 9x^4}\, dx$$

which can be evaluated by the substitution $u = 1 + 9x^4$, $du = 36x^3\, dx$ to get

$$\int_1^1 44 \frac{1}{18} \pi \sqrt{u}\, du = \frac{1}{27}\pi \left(145\sqrt{145} - 1\right)$$

8. (14 points) Solve the initial value problem

$$y' = (2x - 1)(y - 2)$$
$$y(2) = 4$$

Solution:
This is a separable equation so

$$\int \frac{dy}{y-2} = \int (2x-1)\, dx$$

or $\ln|y-2| = x^2 - x + C$ Using the initial condition we get $0 = 2 + C$ and $C = -2$ so

$$y(x) = 2 + \exp\left(x^2 - x - 2\right).$$

9. (15 points)

(a) (5 points) State Simpson's rule for approximating $\int_a^b f(x)\,dx$ using $N = 4$ intervals of size Δx.

Solution:
$\int_a^b f(x)\,dx = \frac{\Delta x}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\right)$

(b) (10 points) The identity

$$\int_1^2 \frac{1}{x}\,dx = \ln(2)$$

gives us a way to compute ln(2) using Simpson's rule. How many intervals N would be required to use Simpson's rule in order to compute ln(2) with an error of no more than 5×10^{-5}? Recall that the error estimate for Simpson's rule applied to $\int_a^b f(x)\,dx$ is

$$\text{Error}(S_N) \le \frac{K_4(b-a)^5}{180N^4}$$

where K_4 is a constant greater than or equal to $f^{(4)}(x)$, the fourth derivative of f, on $[a, b]$.

Solution:
$f(x) = \frac{1}{x}$ so that $f^{(4)}(x) = -24/x^5$ and hence we can choose $K = 24$. Thus we wish to choose N so that

$$\frac{24}{180N^4} \le 5 \times 10^{-5}$$

or

$$N^4 \ge \left(\frac{1}{2} \times 10^5\right) \times \frac{24}{180}$$

or

$$N^4 \ge \frac{20000}{3}.$$

Thus $N = 10$ will do quite nicely.