# *Alternate Exam 1*

Name: ______________________ Section and/or TA: ______________

Last Four Digits of Student ID: ________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used except for a one-page sheet of formulas and facts. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 10 multiple choice questions and 4 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems. Unsupported answers on free response problems will receive *no credit*.

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Multiple Choice Questions

**1** (A) (B) (C) (D) (E)

**2** (A) (B) (C) (D) (E)

**3** (A) (B) (C) (D) (E)

**4** (A) (B) (C) (D) (E)

**5** (A) (B) (C) (D) (E)

**6** (A) (B) (C) (D) (E)

**7** (A) (B) (C) (D) (E)

**8** (A) (B) (C) (D) (E)

**9** (A) (B) (C) (D) (E)

**10** (A) (B) (C) (D) (E)

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SCORE

| Multiple Choice | 11 | 12 | 13 | 14 | Total Score |
|---|---|---|---|---|---|
| 50 | 10 | 10 | 15 | 15 | 100 |
| | | | | | |

Multiple Choice Questions

1. If $\mathbf{a} = \langle 3, -2, 1 \rangle$ and $\mathbf{b} = \langle 1, 0, 2 \rangle$ then $2\mathbf{a} + 3\mathbf{b} =$
   A. $\langle 4, -2, 3 \rangle$
   B. $\langle -4, 2, -3 \rangle$
   **C.** $\langle 9, -4, 8 \rangle$
   D. $\langle -9, 4, -8 \rangle$
   E. $\langle 11, -6, 7 \rangle$

2. What is the distance of the point $(4, 3, 2)$ from the $yz$ plane?
   A. 3
   B. 2
   **C.** 4
   D. 5
   E. $\sqrt{29}$

3. Find the area of the triangle with vertices $Q(1, 0, 2)$, $R(2, 1, 3)$, $S(0, 1, 3)$.
   **A.** $\sqrt{2}$
   B. 1
   C. $2\sqrt{2}$
   D. $2\sqrt{5}$
   E. $\sqrt{5}$

4. Find the equation of a plane perpendicular to the vector $\mathbf{n} = \langle 1, -1, 3 \rangle$ and passing through the point $(1, 2, 3)$.

   A. $x + 2y + 3z = 14$

   B. $x - y + 3z = 9$

   **C.** $x - y + 3z = 8$

   D. $x - y + 3 = -9$

   E. $-x + y + 3z = -8$

5. Which of the following best describes the graph of the equation $z = \left(\frac{x}{4}\right)^2 - \left(\frac{y}{4}\right)^2$?

   A. Ellipse

   B. Elliptic cylinder

   C. Parabolic cylinder

   D. Elliptic paraboloid

   **E. Hyperbolic paraboloid**

6. The function $\mathbf{r}(t) = 3\cos(t)\mathbf{i} + \mathbf{j} + 3\sin(t)\mathbf{k}$ traces out:

   **A. A circle of radius** $3$ **and center** $(0, 1, 0)$ **in the plane** $y = 1$

   B. A circle of radius 3 and center $(0, 1, 0)$ in the plane $x + y = 8$

   C. A circle of radius 3 and center $(1, 0, 0)$ in the plane $x = 1$

   D. A circle of radius 2 and center $(0, 1, 0)$ in the plane $x = 1$

   E. A circle of radius 2 and center $(0, 1, 0)$ in the plane $y = 1$

7. Find the scalar and vector projections of $\mathbf{b} = \langle 4, 6 \rangle$ onto $\mathbf{a} = \langle -5, 12 \rangle$

   **A. Scalar projection $4$, vector projection $\langle -\frac{20}{13}, \frac{48}{13} \rangle$**

   B. Scalar projection $-4$, vector projection $\langle \frac{20}{13}, \frac{-48}{13} \rangle$

   C. Scalar projection $\sqrt{52}$, vector projection $\langle -\frac{5\sqrt{52}}{13}, \frac{12\sqrt{52}}{13} \rangle$

   D. Scalar projection $-\sqrt{52}$, vector projection $\langle \frac{5\sqrt{52}}{13}, -\frac{12\sqrt{52}}{13} \rangle$

   E. Scalar projection 52, vector projection $\langle 260, 624 \rangle$

8. Which of the following best describes the graph of the equation $x^2 + y^2 - z^2 = 1$?

   A. A sphere of radius 1

   B. An elliptic cylinder

   C. A hyperboloid of one sheet with the $x$ axis as an axis of symmetry

   D. A hyperboloid of one sheet with the $y$ axis as axis of symmetry

   **E. A hyperboloid of one sheet with the $z$ axis as axis of symmetry**

9. Consider the planes given by the equations

$$x + 2y - z = 2$$
$$2x - 2y + z = 1$$

   Which one of the following statements is correct?

   A. These planes are parallel

   B. These planes are skew

   **C. These planes intersect one another and the vector $\mathbf{v} = \langle 0, 3, 6 \rangle$ points along the line of intersection**

   D. These planes intersect one another and the vector $\mathbf{v} = \langle 1, 2, -1 \rangle$ points along the line of intersection

   E. These planes intersect one another and the vector $\mathbf{v} = \langle 2, -2, 1 \rangle$ points along the line of intersection

10. Which of the following is *not* a well-defined operation on vectors?

    A. $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$

    **B. $\mathbf{a} \cdot (\mathbf{b} \cdot \mathbf{c})$**

    C. $\mathbf{a} \cdot (\mathbf{a} \times \mathbf{c})$

    D. $(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d})$

    E. $(\mathbf{a} \cdot \mathbf{b})\mathbf{c}$

## Free Response Questions

11. (10 points) Use vectors to decide whether the triangle with vertices $P(1, -3, -2)$, $Q(2, 0, -4)$, and $(6, -2 - 5)$ is right-angled.

**Solution:** Compute

$$\overrightarrow{PQ} = \langle 1, 3, -2 \rangle \quad \text{(2 points)}$$
$$\overrightarrow{PR} = \langle 5, 1, -3 \rangle \quad \text{(2 points)}$$
$$\overrightarrow{QR} = \langle 4, -2, -1 \rangle \quad \text{(2 points)}$$

Compute the dot products (2 points)

$$\overrightarrow{PQ} \cdot \overrightarrow{PR} = 5 + 3 + 6 \neq 0$$
$$\overrightarrow{PR} \cdot \overrightarrow{QR} = 20 - 2 + 3 \neq 0$$
$$\overrightarrow{PQ} \cdot \overrightarrow{QR} = 4 - 6 + 2 = 0$$

Hence $\overrightarrow{PQ}$ and $\overrightarrow{QR}$ are perpendicular, and the triangle is a right triangle. (2 points)

12. (10 points) Use the scalar triple product to determine whether the points $A(1,3,2)$, $B(3,-1,6)$, $C(5,2,0)$, $D(3,6,-4)$ lie in the same plane.

**Solution:** Compute the vectors

$$\overrightarrow{AB} = \langle 2,-4,4\rangle \quad \text{(2 points)}$$
$$\overrightarrow{AC} = \langle 4,-1,-2\rangle \quad \text{(2 points)}$$
$$\overrightarrow{AD} = \langle 2,3,-6\rangle \quad \text{(2 points)}$$

The scalar triple product is

$$\overrightarrow{AB} \cdot \left(\overrightarrow{AC} \times \overrightarrow{AD}\right) = \begin{vmatrix} 2 & -4 & 4 \\ 4 & -1 & -2 \\ 2 & 3 & -6 \end{vmatrix} = 0 \quad \text{(2 points)}$$

so the points are coplanar (2 points)

13. (15 points) (a) (7 points) Find a vector function $\mathbf{r}(t)$ that represents the intersection of the paraboloid $z = 4x^2 + y^2$ and the parabolic cylinder $y = x^2$.

**Solution:** There are several correct solutions–here is one.

Taking $x = t$ as the parameter, we use the equation of the parabolic cylinder to obtain

$$x = t, \quad y = t^2 \quad \text{(2 points)}$$

Next, we use the equation of the paraboloid to conclude that

$$z = 4t^2 + t^4 \quad \text{(2 points)}$$

Putting all the pieces together, we conclude that

$$\mathbf{r}(t) = \langle t, t^2, 4t^2 + t^4 \rangle. \quad \text{(2 points)}$$

Add 1 point because course coordinator can't add and originally made this a 6 point problem

(b) (8 points) Two objects travel through space with trajectories given by the vector functions

$$\mathbf{r}_1(t) = \langle t^2, 7t - 12, t^2 \rangle, \quad \mathbf{r}_2(t) = \langle 4t - 3, t^2, 5t - 6 \rangle.$$

Determine whether these objects collide and, if so, find the coordinates of the collision point.

**Solution:** We seek a value of $t$ which makes the $x$-, $y$- and $z$-components equal. Equating $x$ components we get

$$t^2 - 4t + 3 = 0$$

so that $(t - 3)(t - 1) = 0$. (2 points)

To check for an intersection, we try each of the values $t = 1$ and $t = 3$:

| $t$ | $\mathbf{r}_1(t)$ | $\mathbf{r}_2(t)$ |
|---|---|---|
| 1 | $\langle 1, -5, 1 \rangle$ | $\langle 1, 1, -1 \rangle$ (2 points) |
| 3 | $\langle 9, 9, 9 \rangle$ | $\langle 9, 9, 9 \rangle$ (2 points) |

We conclude that the two particles collide at $t = 3$ and coordinates $\langle 9, 9, 9 \rangle$.

14. (15 points) (a) (6 points) Find the traces of the curve $x^2 - y^2 + z^2 = 1$ in the $xy$, $xz$, and $yz$ planes. In each case, identify the conic section.

**Solution:**
$xy$ plane: $z = 0$ so $x^2 - y^2 = 1$, a hyperbola 1 (2 points)
$xz$ plane: $y = 0$ so the trace is $x^2 + z^2 = 1$, a circle of radius 1 (2 points)
$yz$ plane: $x = 0$ so the trace is $-y^2 + z^2 = 1$, a hyperbola (2 points)

(b) (4 points) Find the center of the quadric surface

$$x^2 - 2x + 4y^2 - 8y + z^2 = 0$$

and identify the quadric surface.

**Solution:** We complete the square to find

$$(x-1)^2 + 4(y-1)^2 + z^2 = 4 \quad \text{(2 points)}$$

so the quadric surface is an ellipsoid with center at $(1, 1, 0)$ (2 points).

(c) (5 points) Using the distance formula, find an equation for the set of all points equidistant from $P(-2, 0, 2)$ and $Q(1, 2, 3)$. Describe the set (e.g. as a line, plane, sphere, ellipsoid, etc.).

**Solution:** Suppose $R(x, y, z)$ is such a point, Then $|PR| = |QR|$ or, using the distance formula

$$\sqrt{(x+2)^2 + y^2 + (z-2)^2} = \sqrt{(x+1)^2 + (y-2)^2 + (z-3)^2}$$
$$x^2 + 4x + 4 + y^2 + z^2 - 4z + 4 = x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 - 6z + 9$$
$$4x - 4z + 8 = 2x - 4y - 6z + 14$$
$$6x + 4y + 2z = 6$$

This is the equation of a plane perpendicular to the vector $\langle 6, 4, 2 \rangle$ It is also OK to use the vector $\langle 3, 2, 1 \rangle$ or another parallel vector here.

Note: This is to be expected since the vector $\overrightarrow{PQ}$ is $\langle 3, 2, 1 \rangle$.

Scoring:

Use distance formula: 2 points
Reduce to linear equation: 1 points

Identify the set as a plane: 2 point