

Quiz 2

Name: _____ Section and/or TA: _____

Answer all questions in a clear and concise manner. Unsupported answers will receive *no credit*.

1. (2 points) Consider the vector $\mathbf{a} = \langle 4, 5, -2 \rangle$ and $\mathbf{b} = \langle 3, -1, 5 \rangle$.

- (a) (1 point) Determine whether \mathbf{a} and \mathbf{b} are orthogonal, parallel, or neither.

Solution: $\cos(\theta) = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|} = \frac{12-5-10}{\sqrt{45}\sqrt{35}} = \frac{-1}{5\sqrt{7}}$, neither.

Alternatively, the dot product non zero tells us they are not orthogonal, the cross product non zero (or they are not proportional) tells us they are not parallel.

- (b) (1 point) What is the angle between \mathbf{a} and \mathbf{b} ? (It is sufficient to leave your answer in the form of \arccos or \cos^{-1}).

Solution: Continue from above, $\theta = \cos^{-1}\left(\frac{-1}{5\sqrt{7}}\right)$.

2. (2 points) Consider the same vectors \mathbf{a} and \mathbf{b} as in the previous question, find a vector that is orthogonal to both \mathbf{a} and \mathbf{b} .

Solution: (Using dot prod) Let (x, y, z) be the vector we want. Then we have $4x + 5y - 2z = 0$ and $3x - y + 5z = 0$ as a system, and one solution is $(23, -26, -19)$.
(Using cross prod) The vector we are looking for is

$$\left(\begin{vmatrix} 5 & -2 \\ -1 & 5 \end{vmatrix} - \begin{vmatrix} 4 & -2 \\ 3 & 5 \end{vmatrix} \begin{vmatrix} 4 & 5 \\ 3 & -1 \end{vmatrix} \right) = (23, -26, -19)$$