Quiz 3

Name: _____ Section and/or TA: _____

Answer all questions in a clear and concise manner. Unsupported answers will receive *no credit*.

1. (2 points) The position vector of a particle is given by

$$\mathbf{r}(t) = \left\langle \frac{1}{2}t^2, 4t, \frac{1}{2}t^2 - 8t \right\rangle$$

(a) (1 point) What is the velocity vector of the particle?

Solution: The velocity vector of the particle can be found by taking the derivative of the position vector:

$$\mathbf{v}(t) = \mathbf{r}'(t) = \langle t, 4, t - 8 \rangle$$

(b) (1 point) What is the speed of the particle?

Solution: The speed of the particle is the magnitude of the velocity vector:

$$s(t) = |\mathbf{v}(t)| = \sqrt{(t)^2 + (4)^2 + (t-8)^2} = \sqrt{2t^2 - 16t + 80}$$

2. (2 points) Using $\mathbf{r}(t)$ from problem 1, when is the minimum speed achieved?

Solution: To find when the speed is minimized we need to find the critical points of $s(t)$. The derivative $s'(t)$ is:

$$s'(t) = \frac{1}{2}(4t - 16)(2t^2 - 16t + 80)^{-1/2} = \frac{(2t-8)}{\sqrt{2}\sqrt{t^2 - 8t + 40}}$$

Since the quadratic $(t^2 - 8t + 40)$ is irreducible (doesn't factor as the product of linear polynomials with real coefficients) $s'(t)$ is always defined. Hence the only critical points occur when $s'(t) = 0$, which is if and only if $2t - 8 = 0$. This occurs when t=4. Since 2t-8<0 for t<4 and 2t-8>0 for t>4, we see that t=4 achieves the minimum speed.