

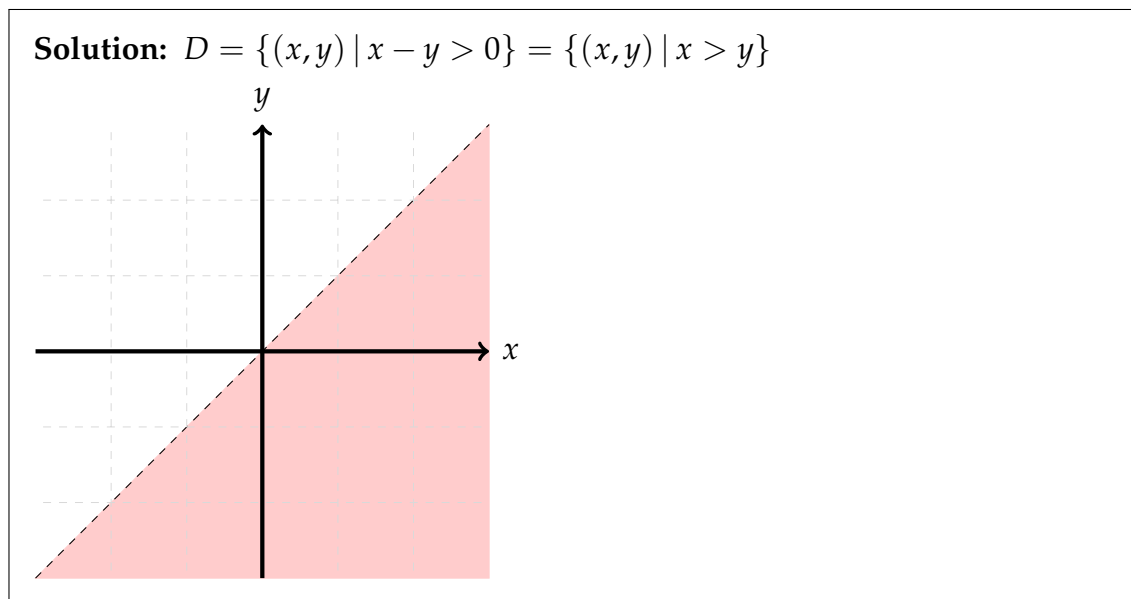
Quiz 4

Name: _____ Section and/or TA: _____

Answer all questions in a clear and concise manner. Unsupported answers will receive *no credit*.

1. (2 points) Let $f(x, y) = \frac{1}{\sqrt{x-y}}$.

(a) (1 point) Sketch the domain of $f(x, y)$.



(b) (1 point) Describe the level curves of $f(x, y)$.

Solution: The level curves of $f(x, y)$ have the form $k = \frac{1}{\sqrt{x-y}}$ or equivalently $y = x - k^{-2}$. Thus the level curves are parallel lines in the xy -plane with slope 1 and y -intercept $-k^{-2}$.

2. (2 points) Let $u(x, y) = \ln(x^2 + y^2)$. Show that $u_{xx} + u_{yy} = 0$.

Solution:

$$\begin{aligned}u_x &= \frac{2x}{x^2 + y^2} & u_y &= \frac{2y}{x^2 + y^2} \\u_{xx} &= \frac{2(x^2 + y^2) - 2x(2x)}{(x^2 + y^2)^2} & u_{yy} &= \frac{2(x^2 + y^2) - 2y(2y)}{x^2 + y^2} \\&= \frac{-2x^2 + 2y^2}{(x^2 + y^2)^2} & &= \frac{2x^2 - 2y^2}{(x^2 + y^2)^2}\end{aligned}$$

Thus,

$$u_{xx} + u_{yy} = \frac{-2x^2 + 2y^2}{(x^2 + y^2)^2} + \frac{2x^2 - 2y^2}{(x^2 + y^2)^2} = 0.$$