Quiz 5

Name: ______________________ Section and/or TA: ______________

Answer all questions in a clear and concise manner. Unsupported answers will receive *no credit*.

1. (2 points) Let $f(x, y) = x\cos(y) - 2xy$.

   (a) (1 point) Find the linear approximation of f at the point $(1, 0, 1)$.

   Solution: The linearization of f at the point $(1, 0, 1)$ is given by $f_x(1, 0)(x - 1) + f_y(1, 0)y + 1$. Taking the x-partial of f, $f_x(x, y) = \cos(y) - 2y$. And taking the y-partial of f, $f_y(x, y) = -x\sin(y) - 2x$. Evaluating each of these at $(1, 0)$ gives $f_x(1, 0) = 1$ and $f_y(1, 0) = -2$. Thus the linearization is $(x - 1) - 2y + 1 = x - 2y$.

   (b) (1 point) Use your answer from part (a) to approximate the number $f(0.9, -0.1)$.

   Solution: We evaluate the linearization, $x - 2y$, at $x = 0.9$ and $y = -0.1$, giving $0.9 - 2(-0.1) = 1.1$.

2. (2 points) Find $\partial z/\partial x$ and $\partial z/\partial y$ assuming z is defined implicitly as a function of x and y as

$$x^3y + 3y^2 - 4z^2 = 0$$

Solution: To find $\partial z/\partial x$, take the x-partial of both sides of the above equation, using the chain rule on z since it is a function of x. This gives $3x^2y - 8z(\partial z/\partial x) = 0$. Now solve for $\partial z/\partial x$,

$$\begin{aligned} 3x^2y - 8z\frac{\partial z}{\partial x} &= 0 \\ -8z\frac{\partial z}{\partial x} &= -3x^2y \\ \frac{\partial z}{\partial x} &= \frac{3x^2y}{8z} \end{aligned}$$

To find $\partial z/\partial y$, similarly take the y-partial of both sides to obtain $x^3 + 6y - 8z\partial z/\partial y = 0$. Solve this for $\partial z/\partial y$,

$$\begin{aligned} x^3 + 6y - 8z\frac{\partial z}{\partial y} &= 0 \\ -8z\frac{\partial z}{\partial y} &= -x^3 - 6y \\ \frac{\partial z}{\partial y} &= \frac{x^3 + 6y}{8z} \end{aligned}$$