Quiz 6

Name: ______________________ Section and/or TA: ______________

Answer all questions in a clear and concise manner. Unsupported answers will receive *no credit*.

1. (2 points) The goal of this question is to compute the double integral

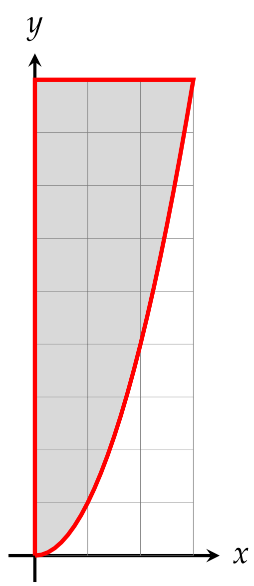
$$\int_0^3 \int_{x^2}^9 x^3 e^{y^3}\,dy\,dx.$$

(a) (1 point) First reverse the order of integration. That is, find a,b, $g_1(x)$ and $g_2(x)$ such that

$$\int_0^3 \int_{x^2}^9 x^3 e^{y^3}\,dy\,dx = \int_a^b \int_{g_1(x)}^{g_2(x)} x^3 e^{y^3}\,dx\,dy$$

Hint: It will help to sketch the region of integration.

Solution:



To reverse the order, we sketch the region bounded by $0 \le x \le 3$ and $x^2 \le y \le 9$. From the sketch, we see that $0 \le y \le 9$ and $0 \le x \le \sqrt{y}$

$$\therefore \int_0^3 \int_{x^2}^9 x^3 e^{y^3}\,dy\,dx = \int_0^9 \int_0^{\sqrt{y}} x^3 e^{y^3}\,dx\,dy$$

(b) (1 point) Use your answer from part (a) to compute the double integral. You should obtain a numerical answer but you don't need to simplify or evaluate with a calculator.

Solution: From part (a), we have

$$\begin{aligned}
\int_0^3 \int_{x^2}^9 x^3 e^{y^3} \, dy \, dx &= \int_0^9 \int_0^{\sqrt{y}} x^3 e^{y^3} \, dx \, dy \\
&= \int_0^9 \left[\frac{x^4}{4} e^{y^3} \right]_{x=0}^{x=\sqrt{y}} dy \\
&= \frac{1}{4} \int_0^9 \left[(\sqrt{y})^4 e^{y^3} - 0 \right] dy \\
&= \frac{1}{4} \int_0^9 y^2 e^{y^3} \, dy \\
&= \left(\frac{1}{4} \right) \left(\frac{1}{3} \right) \left[e^{y^3} \right]_{y=0}^{y=9} \\
&= \frac{1}{12} (e^{729} - 1)
\end{aligned}$$

2. (2 points) Compute the double integral

$$\int_1^2 \int_1^x \frac{x^2}{y^2} \, dy \, dx$$

Solution:

$$\begin{aligned}
\int_1^2 \int_1^x \frac{x^2}{y^2} \, dy \, dx &= \int_1^2 \left[\frac{-x^2}{y} \right]_{y=1}^{y=x} dx \\
&= \int_1^2 \left[\frac{-x^2}{x} \right] - \left[\frac{-x^2}{1} \right] dx \\
&= \int_1^2 \left(-x + x^2 \right) dx \\
&= \left[\frac{-x^2}{2} + \frac{x^3}{3} \right]_1^2 \\
&= \left(\frac{-4}{2} + \frac{8}{3} \right) - \left(\frac{-1}{2} + \frac{1}{3} \right) \\
&= \frac{5}{6}
\end{aligned}$$