Quiz 7

Name: _____ Section and/or TA: _____

Answer all questions in a clear and concise manner. Unsupported answers will receive *no credit*.

1. (2 points) Evaluate the following integral.

   (a) (1 point) Set up the integral as an iterated integral.

   $$\iiint_E 8xyz\,dV$$

   where $E = \{(x, y, z) : 0 \leq x \leq 1,\ 0 \leq y \leq 2,\ 0 \leq z \leq 3\}$.

   Solution:

   $$\int_0^3 \int_0^2 \int_0^1 8xyz\,dx\,dy\,dz$$

   (b) (1 point) Evaluate the integral.

   Solution:

   $$\begin{aligned}
   \int_0^3 \int_0^2 \int_0^1 8xyz\,dx\,dy\,dz &= \int_0^3 \int_0^2 4x^2yz|_{x=0}^{x=1}\,dy\,dz \\
   &= \int_0^3 \int_0^2 4yz\,dy\,dz \\
   &= \int_0^3 2y^2z|_{y=0}^{y=2}\,dz \\
   &= \int_0^3 8z\,dz \\
   &= 4z^2|_{z=0}^{z=3} \\
   &= 36
   \end{aligned}$$

2. (2 points) Evaluate the following integral by using polar coordinates.

   $$\iint_D \cos(x^2 + y^2)\,dA$$

   where D is the region in the first quadrant between $x^2 + y^2 = 1$ and $x^2 + y^2 = 2$.

Solution:

$$\begin{aligned}
\iint_D \cos(x^2+y^2)\,dA &= \int_0^{\frac{\pi}{2}} \int_1^{\sqrt{2}} \cos(r^2) r\,dr\,d\theta \\
&= \int_0^{\frac{\pi}{2}} \frac{1}{2}\sin(r^2)|_{r=1}^{r=\sqrt{2}}\,d\theta \\
&= \int_0^{\frac{\pi}{2}} \frac{1}{2}\Big(\sin(2)-\sin(1)\Big)\,d\theta \\
&= \frac{1}{2}\Big(\sin(2)-\sin(1)\Big)\theta|_{\theta=0}^{\theta=\frac{\pi}{2}} \\
&= \frac{\pi}{4}\Big(\sin(2)-\sin(1)\Big)
\end{aligned}$$